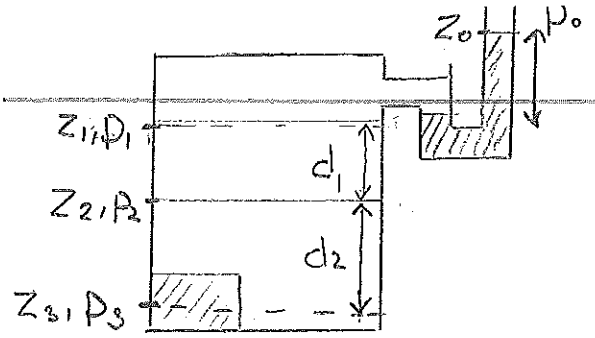
MTF053 strömningsmekanik 110819



Trycket varierar linjärt över plattan; medeltrycket blir då lika med trycket i mitten p_3

$\rho_v = 998\ kg/m^3$ $\quad d_1 = 0{,}6\ m$
$\rho_o = 891\ kg/m^3$ $\quad d_2 = 0{,}65\ m$
$\rho_{Hg} = 13550\ kg/m^3$

$$p_0 - p_1 = -\rho_{Hg}\, g\, (\underbrace{z_0 - z_1}_{h}) \qquad (1)$$

$$p_1 - p_2 = -\rho_o\, g\, (\underbrace{z_1 - z_2}_{d_1}) \qquad (2)$$

$$p_2 - p_3 = -\rho_v\, g\, (\underbrace{z_2 - z_3}_{d_2}) \qquad (3)$$

(1) + (2) + (3) =>

$$p_0 - p_3 = -\rho_{Hg}\, g\, h - \rho_o g d_1 - \rho_v g d_2 \qquad (4)$$

$$h = \frac{p_0 - p_3 + g(\rho_o d_1 + \rho_v d_2)}{-\rho_{Hg}\, g}$$

övertrycket: $p_3 - p_0 = \frac{F}{A}$ (5)

(5) i (4) =>

$$h = \frac{-F/A + g(\rho_o d_1 + \rho_v d_2)}{-\rho_{Hg}\, g}$$

$$= \underline{\underline{0{,}44\ m}}$$

givet: $L = 5\ m$, $b = 1\ m$, $h = 1\ cm$, $F = 850\ N$, oil SAE 50 $\left\{ \begin{array}{l} \rho = 902\ kg/m^3 \\ \mu = 0{,}86\ kg/m\cdot s \\ \nu = 9{,}534\cdot 10^{-4}\ m^2/s \end{array} \right.$

sökt: v

$F = -\tau \cdot A = -\tau \cdot 2 \cdot L \cdot b \Rightarrow \tau = \frac{-F}{2 \cdot L \cdot b} = -85\ N/m^2 \quad \left(= \mu \frac{\partial v}{\partial x}\right)$

N.S. i y-riktn. (endast 2-dim strömning)

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + g_y + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

stationärt: $\frac{\partial v}{\partial t} = 0$; "lång kanal": $u = 0$, $w = 0$, $\frac{\partial v}{\partial y} = 0$, $\frac{\partial v}{\partial z} = 0$, $\frac{\partial^2 v}{\partial y^2} = 0$, $\frac{\partial^2 v}{\partial z^2} = 0$

försumma trycket: $\frac{\partial p}{\partial y} = 0$

$$\Rightarrow 0 = g_y + \nu \frac{\partial^2 v}{\partial x^2} \Rightarrow \frac{\partial^2 v}{\partial x^2} = -\frac{g_y}{\nu}$$

integrera:

$$\frac{\partial v}{\partial x} = -\frac{g_y}{\nu} x + C_1 \qquad (1)$$

integrera:

$$v = -\frac{g_y}{\nu}\frac{x^2}{2} + C_1 x + C_2 \qquad (2)$$

R.V. 1: $x = 0$ $\quad \mu \frac{\partial v}{\partial x} = \tau \Rightarrow \frac{\partial v}{\partial x} = \frac{\tau}{\mu}$

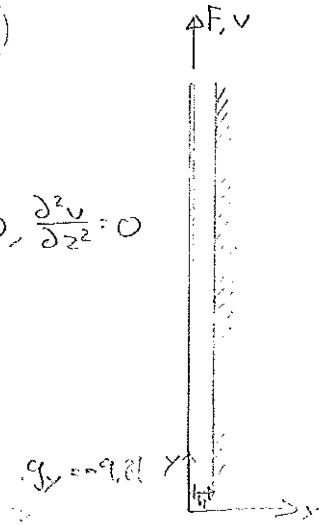
$(1) \Rightarrow \frac{\tau}{\mu} = -\frac{g_y}{\nu}\cdot 0 + C_1 \Rightarrow C_1 = \frac{\tau}{\mu}$

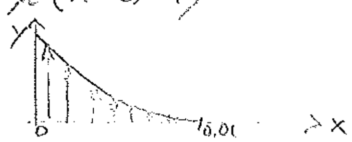
R.V. 2: $x = 0{,}01\ m$ $\quad v = 0$

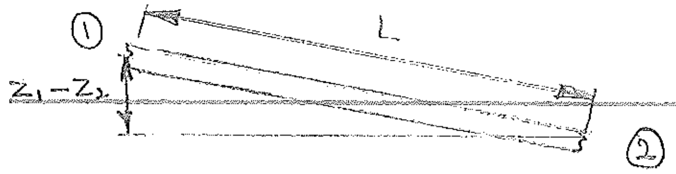
$(2) \Rightarrow 0 = -\frac{g_y}{\nu}\frac{0{,}01^2}{2} + \frac{\tau}{\mu}\cdot 0{,}01 + C_2 \Rightarrow C_2 = \frac{g_y}{\nu}\frac{0{,}01^2}{2} - \frac{\tau}{\mu}\cdot 0{,}01$

$$\Rightarrow v = -\frac{g_y}{\nu}\frac{x^2}{2} + \frac{\tau}{\mu}x + \frac{g_y}{\nu}\frac{0{,}01^2}{2} - \frac{\tau}{\mu}0{,}01 = \frac{g_y}{\nu\cdot 2}(0{,}01^2 - x^2) + \frac{\tau}{\mu}(x - 0{,}01)$$

$v(x=0) = 0{,}47\ m/s$







Givet: $d = 0{,}4$ m $p_1 = 150$ kPa
$L = 320$ m $p_2 = 120$ kPa
$z_1 - z_2 = 4{,}5$ m
$t = 10°C$
$Q_a = 1100\ m^3/h = \frac{11}{36}\ m^3/s$
$Q_b = 1{,}4\, Q_a$

Lösn: Bs utvidgade ekv (3.68b) eller (3.78) eller (6.24)

$$p_1 + \rho \frac{\alpha_1 V_1^2}{2} + \rho g z_1 = p_2 + \rho \frac{\alpha_2 V_2^2}{2} + \rho g z_2 + \Delta p_f$$

$V_1 = V_2$, $\alpha_1 = \alpha_2$

a) $\Delta p_f = p_1 - p_2 + \rho g (z_1 - z_2) = (1{,}5 - 1{,}2)\cdot 10^5 + \rho g \cdot 4{,}5$

Kylvatten, $t = 10°C \Rightarrow \rho = 10^3$ kg/m³
$\nu = 1{,}31 \cdot 10^{-6}$ m²/s

$\Delta p_f = 0{,}3\cdot 10^5 + 10^3 \cdot 9{,}81 \cdot 4{,}5 = 0{,}74 \cdot 10^5$ Pa

(6.30b):

$$\Delta p_f = f \frac{L}{d} \rho \frac{V^2}{2} = f \frac{L}{d} \frac{\rho Q^2 4^2}{2(\pi d^2)^2} \Rightarrow$$

$$f = \Delta p_f \frac{\pi^2 d^5}{8 L \rho Q^2} = \frac{0{,}74\cdot 10^5\ \pi^2\, 0{,}4^5\, 36^2}{8 \cdot 320 \cdot 10^3 \cdot 11^2}$$

$f_a = 0{,}0314$

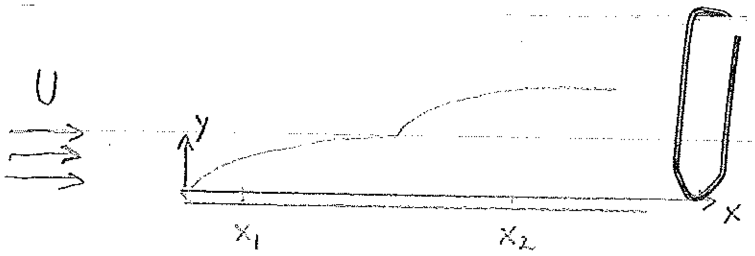
$$Re = \frac{Vd}{\nu} = \frac{Q \cdot d \cdot 4}{\pi d^2 \nu} = 7{,}5 \cdot 10^5$$

Moody-diagram, Fig 6.13, s. 318 $\Rightarrow$ f är oberoende av Re.

b) $\therefore\ f_b = f_a$ enl. ovan

$$\Delta p_f = f_b \frac{L}{d} \rho \frac{V_b^2}{2} = 14{,}6 \text{ kPa}$$

Det räcker alltså inte att öka trycket p_1 till 170 kPa.



Givet: $U = 50$ m/s
$u(x_1, y_1) = u(x_2, y_2) = 42$ m/s
$x_1 = 0{,}05$ m $x_2 = 2{,}5$ m
$t = 20°C$, $p = 100$ kPa $\nu = 15{,}2\cdot 10^{-6}$ m²/s, $\rho = 1{,}189$ kg/m³

Sökt: y_1 och y_2

Losning:

a) $x = x_1$

$$Re_{x_1} = \frac{U x_1}{\nu} = 1{,}64 \cdot 10^5 < Re_{x_{kr}} \Rightarrow \text{lam.}$$

$$\frac{u}{U} = \frac{42}{50} = 0{,}84 \quad \text{Tab. 7.1} \Rightarrow \eta = 2{,}96$$

$$y = \frac{\eta}{\sqrt{\frac{U}{\nu x}}} = 3{,}65 \cdot 10^{-4} \text{ m}$$

b) $x = x_2$ $Re_{x_2} = 8{,}224 \cdot 10^6 > Re_{x_{kr}} = 5 \cdot 10^5$

$\therefore$ turb gs. Antag omslag redan i framkanten

$$(7.43) \Rightarrow \tau_w = 0{,}135 \frac{\rho U^2}{\sqrt[7]{Re_x}} = 4{,}127 \text{ Pa}$$

$$u^* = \sqrt{\frac{\tau_w}{\rho}} = 1{,}863 \text{ m/s}$$

Antag log-omr: $\frac{u}{u^*} = 2{,}44 \ln \frac{u^* y}{\nu} + 4{,}9$

$$\Rightarrow y = \frac{\nu}{u^*} e^{\left(\frac{u}{u^*} - 4{,}9\right)/2{,}44} = 0{,}0112 \text{ m}$$

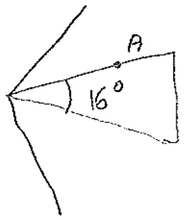
Kontroll:

$\frac{u^* y}{\nu} = 1382$, log-lagen kan anses gälla

Svar a) $y = 0{,}37$ mm b) $y = 11{,}2$ mm

P4 01-01-11

a) $Ma = 3$ $\longrightarrow$ $P = 70\,kPa$



Luft $\Rightarrow k = 1{,}4$

För $Ma_1 = 3$; $\theta = 8°$ ger ekv (9.86)

$$\tan\theta = \frac{2\cot\beta\,(Ma_1^2 \sin^2\beta - 1)}{Ma_1^2\,(k + \cos 2\beta) + 2}$$

Iterering $\Rightarrow \beta = 25{,}61°$

$$\frac{P_A}{P_1} = \frac{1}{k+1}\cdot\left[2k\cdot Ma_1^2 \sin^2\beta - (k-1)\right] \quad (9.83a)$$

$\Rightarrow P_A = 126$ kPa

b) En normal stöt formas

$\Rightarrow P_B = P_{02}$

Tabell B1 $\Rightarrow P_{01} = P/0{,}0272 \approx 2574\,kPa$

Tabell B2 , $Ma = 3 \Rightarrow P_{02}/P_{01} = 0{,}3283$

$P_{02} = P_B = 0{,}3283 \cdot 2574\ kPa = 845\ kPa.$