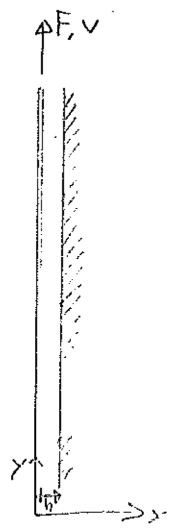
MTF052 Strömningsmekanik 2011-01-12

givet: $L = 5\,m$, $b = 1\,m$, $h = 1\,cm$, $F = 850\,N$, oil SAE 50 $\left\{ \begin{array}{l} \rho = 902\ kg/m^3 \\ \mu = 0{,}86\ kg/m\cdot s \\ \nu = 9{,}534\cdot 10^{-4}\ m^2/s \end{array} \right.$

sökt: v



$$F = -\tau \cdot A = -\tau \cdot 2 \cdot L \cdot b \Rightarrow \tau = \frac{-F}{2 \cdot L \cdot b} = -85\ N/m^2 \quad \left(= \mu \frac{\partial v}{\partial x}\right)$$

N.S. i y-riktn. (endast 2-dim strömning)

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + g_y + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

stationärt: $\frac{\partial v}{\partial t} = 0$; "lång kanal": $u = 0$, $w = 0$, $\frac{\partial v}{\partial y} = 0$, $\frac{\partial v}{\partial z} = 0$, $\frac{\partial^2 v}{\partial y^2} = 0$, $\frac{\partial^2 v}{\partial z^2} = 0$

försumma trycket: $\frac{\partial P}{\partial y} = 0$

$$\Rightarrow 0 = g_y + \nu \frac{\partial^2 v}{\partial x^2} \Rightarrow \frac{\partial^2 v}{\partial x^2} = -\frac{g_y}{\nu}$$

integrera:

$$\frac{\partial v}{\partial x} = -\frac{g_y}{\nu} x + C_1 \qquad (1)$$

integrera:

$$v = -\frac{g_y}{\nu}\frac{x^2}{2} + C_1 x + C_2 \qquad (2)$$

R.V. I: $x = 0$ $\quad \mu \frac{\partial v}{\partial x} = \tau \Rightarrow \frac{\partial v}{\partial x} = \frac{\tau}{\mu}$

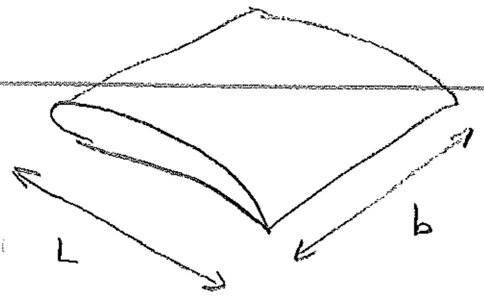
$$(1) \Rightarrow \frac{\tau}{\mu} = -\frac{g_y}{\nu} \cdot \cancel{x}^{\,0} + C_1 \Rightarrow C_1 = \frac{\tau}{\mu}$$

R.V. 2: $x = 0{,}01\ m$ $\quad v = 0$

$$(2) \Rightarrow 0 = -\frac{g_y}{\nu}\frac{0{,}01^2}{2} + \frac{\tau}{\mu} \cdot 0{,}01 + C_2 \Rightarrow C_2 = \frac{g_y}{\nu}\frac{0{,}01^2}{2} - \frac{\tau}{\mu} \cdot 0{,}01$$

$$\Rightarrow v = -\frac{g_y}{\nu}\frac{x^2}{2} + \frac{\tau}{\mu}x + \frac{g_y}{\nu}\frac{0{,}01^2}{2} - \frac{\tau}{\mu}0{,}01 = \frac{g_y}{\nu \cdot 2}(0{,}01^2 - x^2) + \frac{\tau}{\mu}(x - 0{,}01)$$

$$v(x=0) = 0{,}47\ m/s$$



Fullskala	modell
$L = 1{,}0\,m$	$L = 0{,}3\,m$
$U = 25\ m/s$	$U = ?$
$t = 20°C$	$t = 30°C$

D&D eller White:

$\nu = 15{,}2\cdot 10^{-6}\ m^2/s$	$\nu = 16{,}2 \cdot 10^{-6}\ m^2/s$
$\rho = 1{,}189\ kg/m^3$	$\rho = 1{,}151\ kg/m^3$

Sökt $(F_L)_f$

Re-likformighetslag $\Rightarrow (Re)_f = (Re)_m$

$$\left(\frac{UL}{\nu}\right)_f = \left(\frac{UL}{\nu}\right)_m$$

$$U_m = \frac{\left(\frac{UL}{\nu}\right)_f}{(L/\nu)_m} = 88{,}82\ m/s$$

White kap 5 eller utdelade sidor $\Rightarrow F_L = C_L(Re) \cdot A \cdot \rho \frac{U^2}{2}$

Samma Re $\Rightarrow (C_L)_f = (C_L)_m$

$$\frac{(F_L)_f}{(F_L)_m} = \frac{(A\rho U^2)_f}{(A\rho U^2)_m} = \frac{(Lb\rho U^2)_f}{(Lb\rho U^2)_m} =$$

$$= \left[\begin{array}{l}\text{samma skalfaktor} \\ \text{på L och b}\end{array}\right] = \frac{(L^2\rho U^2)_f}{(L^2\rho U^2)_m}$$

$$\Rightarrow (F_L)_f = (F_L)_m \cdot \frac{1{,}0^2 \cdot 1{,}189 \cdot 25^2}{0{,}3^2 \cdot 1{,}151 \cdot 88{,}82^2} \qquad (1)$$

Diagram ger (för $U_m = 88{,}82\ m/s$)

$$(F_L)_m = 4{,}7 \cdot 10^3\ N$$

Ekv (1) $\Rightarrow (F_L)_f = 4274\ N$

Svar: $4{,}3 \cdot 10^3\ N$

Givet: $L = 25\,m$ Olja av $30°C$ $\rho = 980\ kg/m^3$

$d = 0{,}15\ m$

$p_0 = 250\ kPa$ i centrum $\nu = 1{,}0 \cdot 10^{-6}\ m^2/s$

$p = 225\ kPa$

Lösning: $p_0 = p + \frac{\rho w_{mitt}^2}{2} \Rightarrow w_{mitt} = \sqrt{\frac{2(p_0 - p)}{\rho}} = 7{,}14\ m/s$

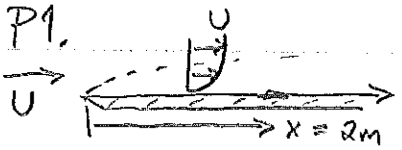
$$Re = \frac{w \cdot d}{\nu} = k \cdot \frac{w_{mitt} \cdot d}{\nu} = k \cdot \frac{7{,}14 \cdot 0{,}15}{10^{-6}} = k \cdot 1{,}07 \cdot 10^6$$

Strömningen är alltså turbulent med $k \approx 0{,}82$

$$\dot m = \rho A w = 980 \cdot \frac{\pi \cdot 0{,}15^2}{4} \cdot 0{,}82 \cdot 7{,}14 = 101\ kg/s$$

Svar: $\dot m = 100\ kg/s$

P1.



Sill I: $\tau_w(x = 2m) = 2{,}1\ Pa$

Luft, $20°C$, 1 atm, [A2] $\Rightarrow \rho = 1{,}20\ \frac{kg}{m^3}$

$\mu = 1{,}80 \cdot 10^{-5}\ Ns/m^2$, $\nu = 1{,}50 \cdot 10^{-5}\ m^2/s$

Sökt: a) U

b) τ_w med uttrycket

$$\frac{\rho y^2 \tau_w}{\mu^2} \approx 0{,}0007 \left(\frac{uy}{\nu}\right)^{1{,}77}$$

om $y = 5\ mm$

LÖSNING: a) Anta att strömningen är turbulent.

(7.44): $\tau_{w,turb} \approx \frac{0{,}0135 \mu^{1/7} \rho^{6/7} U^{13/7}}{x^{1/7}}$

$$\Rightarrow U \approx \left(\frac{\tau_{w,turb}\, x^{1/7}}{0{,}0135 \mu^{1/7} \rho^{6/7}}\right)^{7/13} \approx 34{,}0\ m/s$$

Kontroll om antagandet om turbulent strömning stämmer!

$$Re_x = \frac{Ux}{\nu} = 4{,}5 \cdot 10^6 > Re_{x,kr} \approx 3 \cdot 10^6$$

Alltså OK med turb. strömning.

b) Har alltså turb. strömn., $y = 5\ mm$

Anta att vi befinner oss i log-omr.

(6.21) $\frac{u}{u^*} = \frac{1}{\kappa} \ln\left(\frac{y u^*}{\nu}\right) + B$, $\begin{cases} \kappa = 0{,}41 \\ B = 5{,}0 \end{cases}$

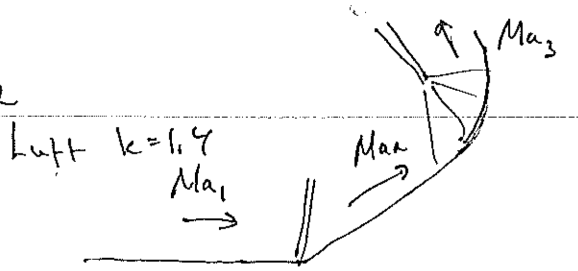
Med $y = 5\ mm$ och $u^* = \sqrt{\tau_w/\rho} \approx 1{,}32\ m/s$

fås $u = u^* \left\{\frac{1}{\kappa} \ln\left(\frac{y u^*}{\nu}\right) + B\right\} \approx 26{,}3\ m/s$

Kontroll: $y^+ = \frac{y u^*}{\nu} = 441$, alltså OK med log-lagen.

Svar: a) $U = 34\ m/s$ b) $u = 26{,}3\ m/s$

P2



Efter stöten komprimeras luften när väggen med en Ma-fana till $Ma_3 = 1$

Sned stöt med $\beta = 40°$

$Ma_1 = 3$

$Ma_{1n} = Ma_1 \cdot \sin\beta = \ldots = 1{,}92836$

Räkna över stöt

B3f)

$$Ma_{2n}^2 = \frac{(k-1)\,Ma_{1n}^2 + 2}{2k\,Ma_{1n}^2 - (k-1)} = 0{,}3483$$

$\Rightarrow Ma_{2n} = 0{,}5901$

$(9.86) \Rightarrow \tan\theta = f(\beta, Ma_1)$

$\Rightarrow \theta = 21{,}8461$

$$Ma_2 = \frac{Ma_{2n}}{\sin(\beta - \theta)} = 1{,}893953$$

Läs av $\omega(Ma_2)$ i tabell B5

$\Rightarrow \omega(Ma_2) = 23{,}417$

Eftersom $\omega(Ma=1) = 0$ så blir vinkeln

$\Delta\omega = 0 - 23{,}417 \approx -23{,}4°$