Re-exam VSM167 FEM – Basic, 2020-04-08

Time: 14:00-18:00 (+ additional 30 min to upload Part 2 to Canvas)

All aids were allowed.

The exam was conducted on Canvas. It had two parts.

Part 1: Quiz – 4p

Students were given 5 out of the 6 quiz questions in a randomized order. Each question could give a maximum of 0.8p

Part 2: Problem solving – 5p

This part consisted of Problems 1-4 as they are defined below

In total, the exam could give 9 points, and the standard grading scale was used.

Exam quiz questions

Remember, changes to question templates won't automatically update quizzes that are already using those questions.

Show question details

Convergence criteria 0.8 pts

Explain the two main requirements on the element approximations (i.e. on the element shape functions) to guarantee the convergence of results from a finite element simulation.

If/when you introduce any concepts (or terminology), please be careful to explain exactly what they mean. 1-3 sentences per concept should be enough, but fell free to write longer (but consider the exam time available)

move/copy question to another bank

Convergence in results 0.8 pts

skip this question

Consider the solution of a one-dimensional heat problem $T_h(x)$ solved by the use of second order approximation functions (quadratic shape functions). For an element size of h, the error of the solution (the value of the simulated temperature T_h vs. the correct value of the temperature T) at a point of interest is given by

$$e(h) = |T - T_h|$$

What is theoretically the error if the element size is reduced to $h/3$?

Consider in this case that the problem is such that the order of the variation of T with respect to x is higher than that of the approximation function.

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Improving the results 0.8 pts

Consider that you are working as an engineer and that you have just received the results from a plane strain 2D analysis of the stress distribution in a rectangular AISI 316 steel plate with a hole. The simulation was run as a 2D elasticity solution using material parameters from the literature.

The dimensions of the plate are length 100 mm, width 50 mm, hole diameter 10 mm (placed in the centre) and plate thickness of 10 mm. The analysis was run with linear triangular elements with an approximate size of 5 mm (length of an element edge).

You are interested in higher accuracy of you results than the ones you have just obtained. **Give at least 3 ways for how you can achieve an increased accuracy in the stress predictions compared to what you have just obtained.** Short statements for each way of improving the solution are enough.

(wrong suggestions may lead to point deduction)

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Partitioning for prescribed degrees of freedom 0.8 pts

Consider the truss problem in the figure below to be solved given the prescribed displacements as shown by classical boundary condition symbols (at the bottom) and the prescribed displacement arrow (u).

In general, such a problem can be partitioned into free and prescribed degrees of freedom such that:

$$\begin{bmatrix} \mathbf{K}_{ff} & \mathbf{K}_{fp} \\ \mathbf{K}_{pf} & \mathbf{K}_{pp} \end{bmatrix} \begin{bmatrix} \mathbf{a}_f \\ \mathbf{a}_p \end{bmatrix} = \begin{bmatrix} \mathbf{f}_f \\ \mathbf{f}_p \end{bmatrix}$$

where $\mathbf{K}_{ff}$ etc. are submatrices of $\mathbf{K}$, $\mathbf{a}_f$and $\mathbf{a}_p$ are subvectors of $\mathbf{a}$ (the vector with all degrees of freedom) and $\mathbf{f}_f$and $\mathbf{f}_p$ are subvectors of the load vector.

Consider the matrix notation such the the first index indicate the row and the second index indicate the column. For this notation, a 3 x 3 matrix would be written as:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

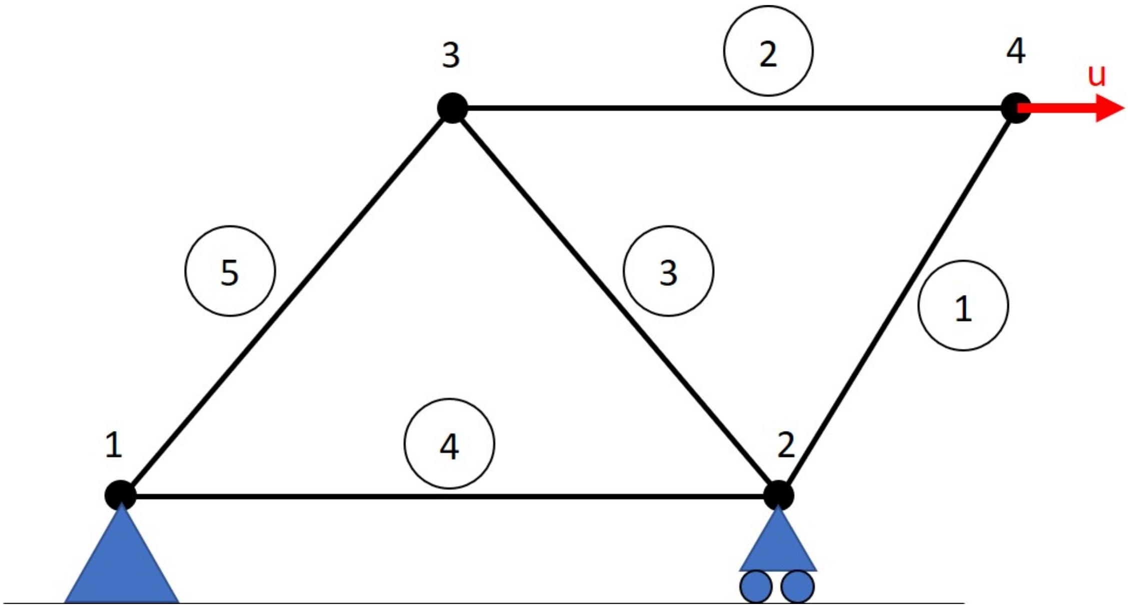
or as

K = [K11, K12, K13; K21, K22, K23; K31, K32, K33];

in MATLAB format.

Define a numbering scheme of the degrees of freedom as a function of node numbers and then define the matrix $\mathbf{K}_{pf}$ for this partitioning. Start from assuming that the global stiffness matrix $\mathbf{K}$ is given with components K_{ij} where index i denotes the row and j denotews the coloumn (as examplified with the 3 x 3 matrix above). Do not derive or compute any stiffness contributions!

Feel free to write the answer in MATLAB (or any other) notation as long as you specify what you mean.



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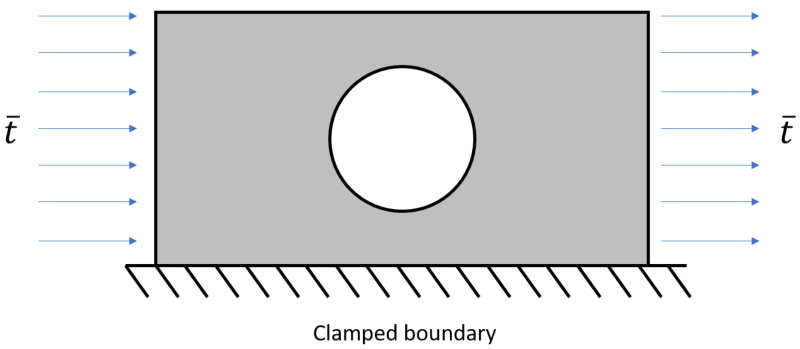
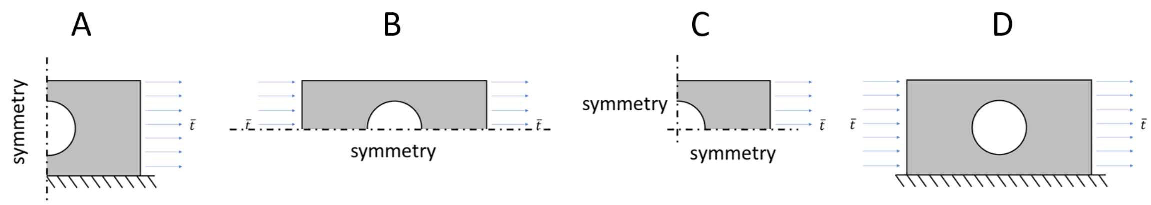
Symmetry

0.8 pts

Consider the 2D elasticity problem in the figure below.

Identify the smallest possible domain that can be analysed to solve the problem by choosing the appropriate subfigure in the second picture below (A, B, C or D). Motivate your answer with a few sentences.





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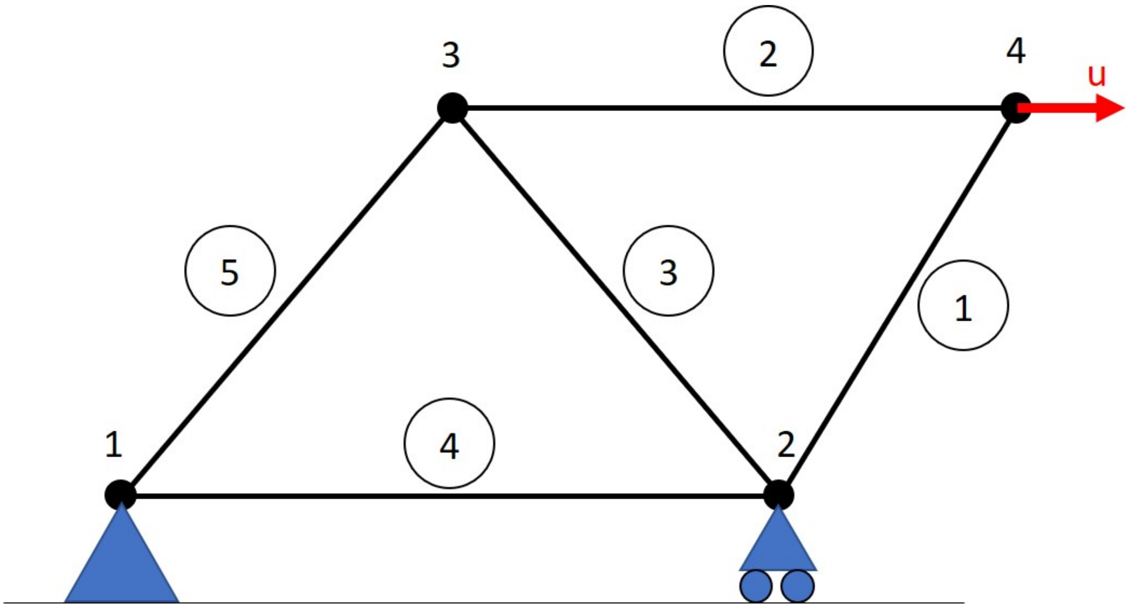
Topology

0.8 pts

Consider the following truss structure to be analysed by the finite element method. Based on the element and node numbering given in the figure, think about how to construct a matrix that can be used to assemble the element stiffness matrices (4 x 4 matrices) into the global stiffness matrix.

Provide a short explanation for how you suggest to define the matrix to be used for the assembly of the stiffness matrix. Specifically, **also write the two rows (or coloumns depending on how you construct the matrix) associated with element 2 and 4**. Use MATLAB-format and write directly in the text field below.



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Problem 1

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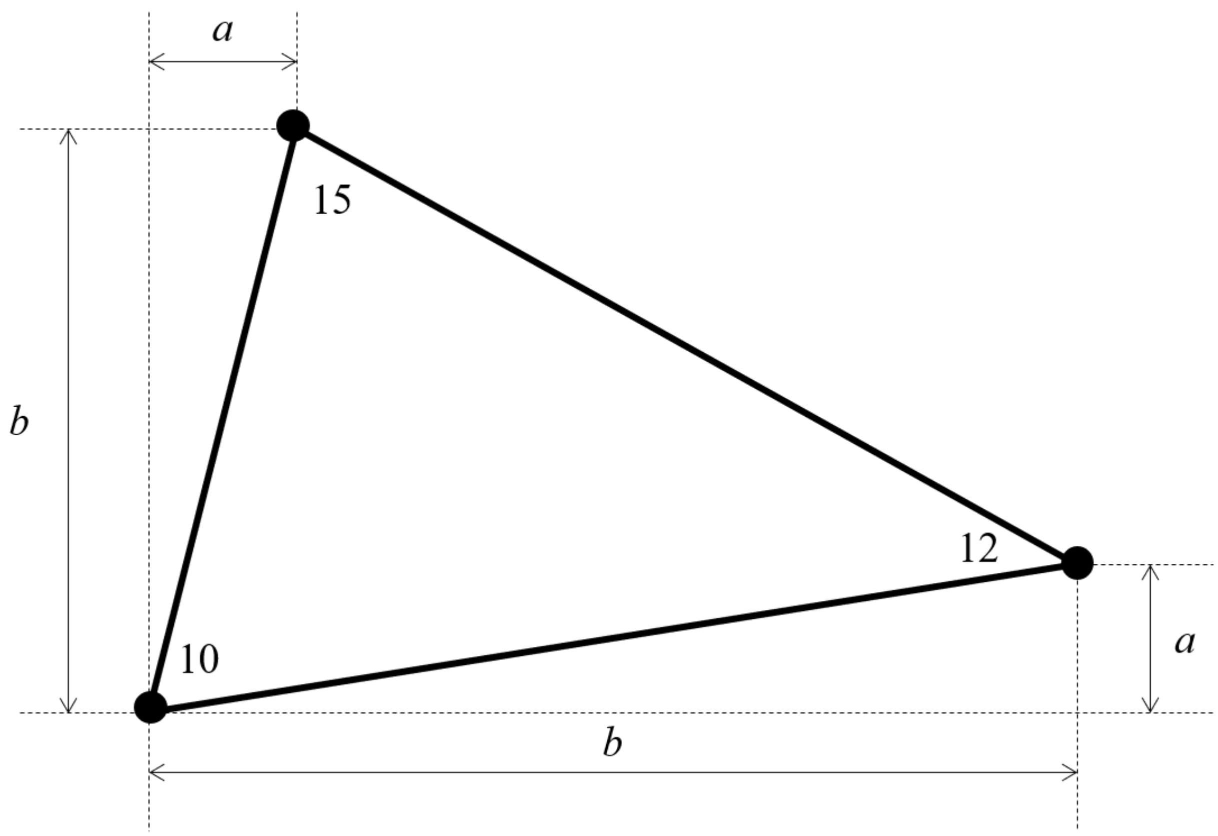
Consider a 3-noded linear triangular element as exemplified in the figure below, used to solve a transient heat transfer problem. At the time instant t [redacted], the values of the degrees of freedom in the three nodes are $T_{10} = 10\,^\circ C$, $T_{12} = 15\,^\circ C$ and $T_{15} = 20\,^\circ C$. **Calculate, for this time instant, the gradient of the temperature field and how this varies within the element.**

Again, the geometry of the element is given by coefficients *a* = A+C mm and *b*=B+D mm (please note the difference from other tasks) which again are to be calculated from the day you were born

A: first digit in the month you are born
B: second digit in the month you are born
C: first digit in the day you are born
D: second digit in the day you are born

Please note that for some dates of birth, the nodes may change place (compared to the figure below). In such a case, be extra careful in defining your element geometry with a sketch (including node numbers and their position). For all cases, be extra careful to clearly state your date of birth.



Points	1.5
Submitting	a file upload
File types	doc, docx, and pdf

Due	For	Available from	Until
8 Apr at 18:30	VSM167 2020-04-08	8 Apr at 14:00	8 Apr at 18:30

+ **Rubric**

Problem 2

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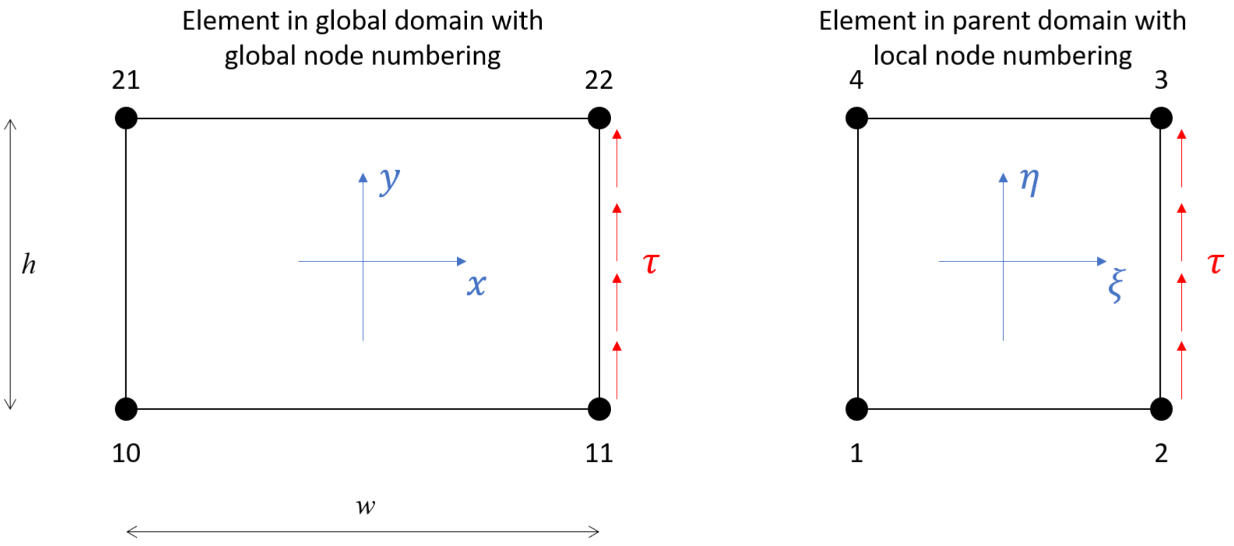
Consider a bilinear quadrilateral element (4 nodes) used in a 2D elasticity problem subjected to a constant shear traction τ = 10 MPa along one of its edges, as shown in the figure below.

The height of the element is $h = A + B$ mm, and the width of the element is $w = C + D$ mm where $A - D$ are coefficients that you find from your personal number in the following way:

A: first digit in the month you are born
B: second digit in the month you are born
C: first digit in the day you are born
D: second digit in the day you are born

Calculate the contribution to the global load vector from the applied shear traction, including both **the values of the load components** and to which p**osition in the global load vector the different contributions are to be added**.

Please be extra careful to clearly state your date of beith along with the solution.



Points	1
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Problem 3

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Consider the polynomial:

$$f = Ax^3 + Bx^2 + Cx + D$$

where $A - D$ are coefficients that you find from your personal number in the following way:

A: first digit in the month you are born
B: second digit in the month you are born
C: first digit in the day you are born
D: second digit in the day you are born

As an example, for a person born 15 April, the polynomial becomes:

$$f = 4x^2 + x + 5$$

Use numerical integration to integrate the function exactly over the interval $0 \leq x \leq 2$ (integrate from 0 to 2).

Please be careful to state your date of birth along with the solution.

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+ Rubric

Problem 4

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Define an example finite element problem of one type that has been analysed in the course (heat transfer, vapour diffusion, elasticity, ...) . Please note that your example problem may be similar but not identical to the examples in the assignments or on the regular exam.

Given the example problem you have define, state its strong form.

Then, **describe the solution procedure necessary for you to determine the quantity of interest using a small finite element program that you implement on your own** (Note! You are not supposed to implement anything, just describe the steps required).

Your description of the solution procedure should include:

1. A list of main derivation and calculation steps to get from the strong form to the result you are interested in (just list the steps with a very short explanation, there is no need to do any derivations or calculations)
2. The key parts of your finite element programme

To complete the task well, please be specific in how you define the original problem as well as what result you are interested in. You may of course write by hand or using any software. In the end, please produce a pdf with text and a sketch of your problem. It should be enough with a maximum of 1 page of regular text, and may be longer if you use figures, flowcharts and/or equations.

Points	1
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+ Rubric

Problem 1:

- Correct element area: 0.3p
- Correct shape function derivatives: 0.7p
- Correct use of grad(T) = Ba: 0.3p
- Correct unit in the answer: 0.2p

Problem 2:

- Correct definition of traction forces: 0.3p
- Correct integration of the traction: 0.3p
- Correct assembly: 0.2p
- Correct unit in the answer: 0.2p

Problem 3:

- Correct integration scheme with motivation: 0.4p
- Correct locations of integration points: 0.4p
- Correct weights: 0.2p
- Clear description of integration rule: 0.2p
- Correct evaluation: 0.3p

Problem 4:

- Sketch of problem: -0.2p if missing
- Strong form: 0.2p (0.1 for eq + 0.1p for BCs)
- Solution procedure
    - Main derivation steps – 0.4p in total
        - Derive weak form 0.2p
        - Derive FE form 0.2p
    - Main calculation steps – 0.4p in total
        - Definition of discretization 0.1p
        - Calculation of element contributions 0.1p
        - Assembly of element contributions 0.1p
        - Calculation of additional loads if applicable -0.1p
        - Solution of system of equations with consideration of prescribed dofs (if applicable) 0.1p