Reexam for the course LMA017 Mathematical Analysis in Several Variables

Date: January 3, 2022
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The reexam is 50 points in total to collect. Grade limits:

20 - 29 points for the grade 3,

30 - 39 points for the grade 4,

40 or more points for the grade 5.

(Bonus points collected during the course will be added).

Allowed aids: one can bring up to 3 pieces of paper of size A4 with anything written or printed on it. Both sides of paper can be used. **No electronics is allowed!**

Arguments should be presented in full. Only providing an answer will normally not be rewarded with points.

The reexam consists of 7 problems. They are distributed over 2 pages.

Good luck!

Problem 1 (4 PTS)

Find the limit, if it exists, or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+2y^5}$$

Problem 2 (10 PTS)

Let $f(x,y) = x^2 + 2y^2 - 2x - 7$.

(a) (4 pts) Find and classify critical points of f.

(b) (6 pts) Find the absolute maximum and minimum values of f on the region

$$D = \{(x,y) \mid x^2 + y^2 \leq 4\}.$$

Problem 3 (6 PTS)

Compute the volume of the solid that lies below the paraboloid $z = 9 - x^2 - y^2$ and above the xy-plane.

Problem 4 (6 PTS)

Compute the triple integral

$$\iiint_E y \, dV,$$

where E lies under the plane $z = 1 + 2x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 1$, $x = 0$.

Problem 5 (10 PTS)

Let C be the curve given by the parametrization

$$\bar{r}(t) = 2\cos t\,\bar{i} + (1-t)\,\bar{j} + 2\sin t\,\bar{k}, \quad 0 \le t \le \pi.$$

(a) (4 pts) Find the length of C.

(b) (6 pts) Find the work done by the force field

$$\bar{F} = x\,\bar{i} + y^2\,\bar{j} + z\,\bar{k}$$

in moving a particle along C.

Problem 6 (7 PTS)

Compute the line integral

$$\int_C (x^3 \sin x - xy)\,dx + (2y - e^{y^3-5})\,dy,$$

where C is the curve consisting of the line segments going from $(0,0)$ to $(2,0)$ and from $(2,0)$ to $(1,1)$, and of the arc of the parabola $y = x^2$ from $(1,1)$ to $(0,0)$.

Problem 7 (7 PTS)

Let E be the solid tetrahedron enclosed by the coordinate planes and the plane $2x + 2y + z = 2$. Let S be its surface given with the positive orientation. Let

$$\bar{F} = y^2\,\bar{i} + xz\,\bar{j} + xz\,\bar{k}.$$

Find the flux of $\bar{F}$ across S, that is find

$$\iint_S \bar{F} \bullet d\bar{S}.$$