Exam for the course LMA017 Mathematical Analysis in Several Variables

Date: October 26, 2021, 14:00
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The exam is 50 points in total to collect. Grade limits:

20 - 29 points for the grade 3,

30 - 39 points for the grade 4,

40 or more points for the grade 5.

(Bonus points collected during the course will be added).

Allowed aids: one can bring up to 3 pieces of paper of size A4 with anything written or printed on it. Both sides of paper can be used. **No electronics is allowed!**

Arguments should be presented in full. Only providing an answer will normally not be rewarded with points.

The exam consists of 7 problems. They are distributed over 2 pages.

Good luck!

Problem 1 (4 PTS)

Find the limit, if it exists, or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2+3xy+y^2}{x^2+y^2}$$

Problem 2 (10 PTS)

Let $f(x,y) = xy$.

(a) (4 pts) Find and classify critical points of f.

(b) (6 pts) Find the absolute maximum and minimum values of f on the region

$$D = \{(x,y) \mid 9x^2+y^2 \leq 18\}.$$

Problem 3 (6 PTS)

Compute the double integral

$$\iint_D y(x^2+y^2)^{3/2}\, dA,$$

where D is the region that lies in the first quadrant (i.e. $x, y \geq 0$) between the circles $x^2+y^2=1$ and $x^2+y^2=16$.

Problem 4 (6 PTS)

Compute the volume of the solid

$$E = \{(x,y,z) \mid 0 \leq y \leq 1,\ y^2 \leq x \leq 1,\ -1 \leq z \leq xy\}.$$

Problem 5 (10 PTS)

Let C be the curve given by the parametrization

$$\bar{r}(t) = 3\sin t\ \bar{i} + 3\cos t\ \bar{j} + 4t\ \bar{k}, \quad 0 \leq t \leq \pi.$$

(a) (4 pts) Find the length of C.

(b) (6 pts) Find the work done by the force field

$$\bar{F} = -y\ \bar{i} + x\ \bar{j} + z\ \bar{k}$$

in moving a particle along C.

Problem 6 (7 PTS)

Compute the line integral

$$\int_C (x^2y + e^{x^2})\ dx + (y^3 - \sin(e^y))\ dy,$$

where C is the trapezoidal curve consisting of the line segments going from $(0,0)$ to $(1,0)$, from $(1,0)$ to $(1,2)$, from $(1,2)$ to $(0,1)$, and from $(0,1)$ to $(0,0)$.

Problem 7 (7 PTS)

Let

$$E = \{(x,y,z) \mid 0 \leq x \leq 1,\ 0 \leq y \leq 2,\ 0 \leq z \leq 3\}$$

and let S be its surface given with the positive orientation. Let

$$\bar{F} = xy\ \bar{i} + xe^z\ \bar{j} + xe^y\ \bar{k}.$$

Find the flux of F across S, that is find

$$\int_S \bar{F} \bullet d\bar{S}.$$