

Tentamen LMA017

Mathematical sciences, Chalmers University of Technology

Date:	January 4th 2021, 08.30.
Examiner:	Axel Flinth
Allowed aids:	Any.
Grade limits:	20 points for the grade 3. 30 points for the grade 4 40 points for the grade 5.

There are in total 50 points to collect.

Calculations and arguments should be presented in full. **Only providing an answer will normally not be rewarded with points.** Solutions may be written in Swedish or English (or German).

If you use any external tool or resource, you should reference it. See [the Canvas Page](#) for more information. Not referencing properly may result in point deduction.

The exam consists of **seven (7)** problems. They are distributed over **three (3)** sheets.

Good luck!

Problem 1 (6p)

Let $g : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined through

$$g(x, y) = \begin{bmatrix} x^2y \\ y \sin(x) \\ x \end{bmatrix}.$$

1. Calculate the derivative of g . (3p)
2. Evaluate g and g' in $\langle 1, 0 \rangle$. (3p)

Problem 2 (7p)

Calculate the center of mass of the triangle with corners $\langle 0, 0 \rangle$, $\langle 0, 1 \rangle$ and $\langle 1, -1 \rangle$.

Problem 3 (6p)

Calculate $\iiint_S z dx dy dz$, where S is the part of the unit ball that lies above the xy -plane.

Problem 4**(6p)**

Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined through

$$f(x, y) = \cos\left(\frac{\pi}{2}x\right) \cos\left(\frac{\pi}{2}y\right).$$

Determine the global maximum and minimal values of f on the square $[-1, 1]^2$.

Problem 5**(7p)**

Milk is poured into a mug of coffee. The amount of coffee lies between 2.3 and 2.7 deciliters, and the amount of milk is somewhere between 0.3 and 0.7 deciliters. Use the Schrankensatz to give an estimate of the fraction of the liquid in the cup that is milk, with error bounds.

Problem 6**(6p)**

Let $\mathbf{w} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the vector field

$$\mathbf{w}(x, y, z) = \begin{bmatrix} xy^2 + z \\ yz^2 + x \\ zx^2 + y \end{bmatrix}.$$

Let further K be the cube $[0, 1]^3 = \{|x, y, z| 0 \leq x, y, z \leq 1\}$, and S its surface. Calculate the flow of \mathbf{w} out of K , i.e.

$$\iint_S \mathbf{w} \cdot d\mathbf{S}.$$

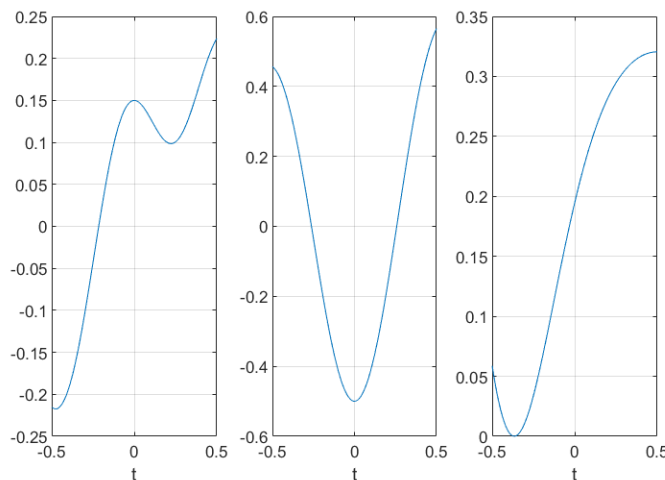
Problem 7**(6p)**

A ball is rolling on the graph of a twice differentiable function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$. The $\langle x, y \rangle$ -coordinates of the ball follow a curve $\gamma(t)$. At $t = 0$, these coordinates reside in the point $\gamma(0) = \langle 0, 0 \rangle$, and the velocity of them is $\gamma'(0) = \langle 1, 2 \rangle$. We know that

$$\nabla f(0, 0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad f''(0, 0) = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}.$$

We let g denote the function which describes how the height of the ball depends on time, i.e. $g = f \circ \gamma$.

1. What is the derivative of g in 0, i.e. $g'(0)$? (2p)
2. What is the second derivative of g in 0, i.e. $g''(0)$? (2p)
3. Which of the following graphs depicts g ? (2p)



Only ruling out one graph with a good justification will be awarded one point.

Problem 8**(6p)**

Let $\mathbf{v} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the vector field

$$\mathbf{v}(x, y) = \begin{bmatrix} -y \\ x \end{bmatrix}$$

- (a) Determine the line integral of \mathbf{v} along the unit circle. You can choose which direction the circle is traversed. (2p)
- (b) Does there exist a differentiable function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ with the property that

$$f(\mathbf{p}) = 1 \text{ for all } \mathbf{p} \text{ in the unit circle}$$

so that $\mathbf{w}(\mathbf{p}) = f(\mathbf{p})\mathbf{v}(\mathbf{p})$ is conservative? (4p)