

Exam QM (IEK312) – 29th October 2016

Errata

In the multiple-choice question Q7 (DoE) where it is written:

$$|l_C| > |l_{BC}| \text{ and } |l_C| > 0 ; |l_{BC}| < 0$$

It should be:

$$|l_C| > |l_{BC}| \text{ and } l_C > 0 ; l_{BC} < 0$$



EXAMINATION

Quality Management

IEK312

Examiner:	Marco Santos, 070 403 94 38
For questions:	Marco Santos will visit the exam place one hour after the beginning and one hour before the end of the examination.
Aids at the exam:	Non-programmable calculator, dictionary
Grading scale:	Grade 3: 20p-29p / Grade 4: 30p-39p / Grade 5: 40p-50p (Max 50p)
Results:	The students will receive an email within three weeks after the exam.
Grading review:	23 rd November 2016, 11:00-12:30 at Marco Santos' office at the Division of Service Management and Logistics (floor 3, building opposite to Vasa C)

OBSERVE

New answer sheet for each question: Start each question with a new answer sheet. Write the question number and your code on top of each page.

Relevance: Irrelevant answers can give reduced amount of points.

Handwriting: Write legibly! Illegible answers will be disregarded.

EXAM STRUCTURE

The exam consists of parts 1 and 2, corresponding to 30 and 20 points, respectively. Part 1 consists of six five-point questions. Part 2 deals only with DoE and SPC and includes 20 multiple-choice questions. The exam is to be completed in four hours. The highest grade of the mid-term and part 2 will be used in the calculation of the final grade. The bonus points collected during the course will be added to the number of points in the exam.

PART 1 (30p)

Q1	a) Discuss the <i>relations</i> between the different values (also referred to as principles or cornerstones) within Quality Management. Discuss the relations within 5 pairs of values. Max 1 page	5p
Q2	a) What characterizes a process? b) Discuss how flowcharting can be used to streamline and improve processes. c) Provide a flowchart consisting of at least 8 activities/steps and two roles for the process: "To take a university course" Max 1 page	5p
Q3	a) Describe the purpose of the EFQM Excellence Model b) Explain six of EFQM Excellence Model criteria c) Compare the values on which the EFQM Excellence Model is based to the values in the Cornerstone Model. Max 1 page	5p
Q4	a) Describe each phase of the PDCA-cycle briefly b) Use the PDCA-cycle to analyze your work to prepare for this exam. What were the strengths and weaknesses of your own work in a PDCA-perspective? Max 1 page	5p
Q5	a) Describe the Kano Model and its dynamics. b) Choose a product or a service, ex. mobile telephone or restaurant visit, and apply the Kano Model to the selected product or service. Max 1 page	5p
Q6	a) Explain the concept of robustness and the reason of its importance Answer only one of the sub-questions below: b) Explain the pendulum example in the context of robustness c) Assume that the equation below was obtained from an experiment. The equation allows estimating how an important quality characteristic y is affected by the control factors A , B and C and the noise factor z . How can you use the equation below to get a robust product? Equation: $y = 17 + 3A + 4B + 13C - 3z + 3Az$ Max 1 page	5p

PART 2 (20p)

Instructions for answering the multiple-choice questions

At the end of the exam, you will find the formulas necessary for you to answer the SPC and DoE questions. Please note that you don't need to hand-in this sheet. You should answer the SPC and DoE questions on the sheet with the questions, i.e. "Answer sheet: SPC" and "Answer sheet: DoE". Answers to SPC and DoE questions given elsewhere than on the designated answer sheets will not be considered.

On each answer sheet you will find 10 multiple-choice questions. Please note that more several alternatives may be partially correct. Your task is to select the most correct alternative in each question.

Each correct answer is awarded with 1p whereas **incorrect answers are punished with reduction of 0.33p**. You can receive a maximum of 10p and a minimum of 0p per answer sheet.

Different templates have been created for the entire class by changing the order of questions and/or alternatives. Please note that cheating is not allowed!

You should mark only one alternative per question. Mark your answer with "X" in the square next to your answer. A shaded square is equivalent to leaving it blank. If you write more than one "X" per question, your answer will be considered invalid. Invalid answers will receive 0p. Below follows examples of valid and invalid answers. If you want to mark a certain alternative that you have previously shaded, write a comment in the designated field at the end of the answer sheet.

abc

abc

abc

abc

Valid answer

abc

abc

abc

abc

Invalid answer

abc

abc

abc

abc

Invalid answer

abc

abc

abc

abc

Valid answer

Some formulas for answering the SPC questions

Estimate of process variance	$\hat{\sigma}_x = \frac{\bar{R}}{d_2}$	$\hat{\sigma}_x = \frac{\bar{s}}{c_4}$	$\Phi(z)$ when $N(0,1)$
			z $\Phi(z)$
			3,0 0,9986501019684
			3,5 0,9997673709210
Capability Indices:	$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}_x}$	$\hat{C}_{pk} = \min\left(\frac{USL - \hat{\mu}_x}{3\hat{\sigma}_x}, \frac{\hat{\mu}_x - LSL}{3\hat{\sigma}_x}\right)$	4,0 0,9999683287582
			4,5 0,9999966023269
			5,0 0,9999997133484
			5,5 0,9999999810104
Xbar-chart:	$\mu_x \pm 3 \frac{\sigma_x}{\sqrt{n}}$		6,0 0,999999990134
			6,5 0,999999999598
			7,0 0,999999999987
p-chart:	$p \pm 3 \sqrt{\frac{p(1-p)}{n}}$		Standardization of normal random variables
c-chart:	$c \pm 3\sqrt{c}$		$Z = \frac{x - \mu}{\sigma}$

CHALMERS	Anonymous code	Points for question (to be filled in by teacher)	Consecutive page no. Löpande sid nr
	Anonym kod	Poäng på uppgiften (fylls av lärare)	Question no. Uppgift nr

Template A

Answer Sheet SPC

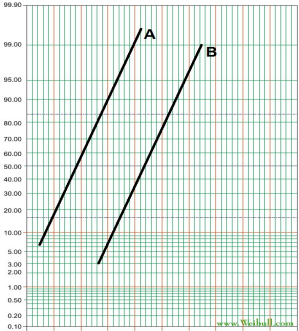
Max: 10p
Min: 0p

Q1 A process that is **not** stable is characterized by:

- A Constant process mean over time
- B Constant process mean and variance over time
- C Freedom from assignable causes of variation
- D Inability to predict the process' future performance

Note: Do not forget to fill in the exam code on this sheet!

Q2 The picture attached shows the theoretical distribution of the individual units produced at two production lines, A and B, using a normal probability plot. What can you say about the capability of the processes A and B assuming that the tolerance limits are the same for both processes?

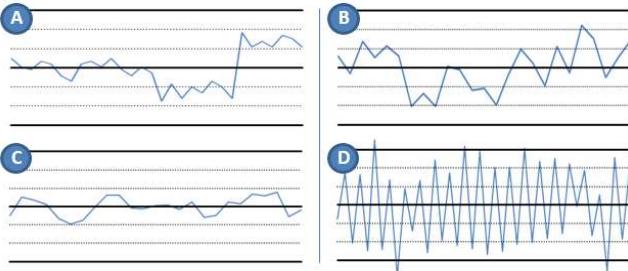


- A The C_p value is similar for A and B
- B The C_{pk} value is similar for A and B
- C The process mean is similar for A and B
- D The FNC (fraction non-conforming) is similar for A and B

Q3 In a nursing home, the weekly number of falls among elders is to be monitored by means of a control chart. Which chart should be used? Note that each elder can fall more than once in a week and that the number of elders in the nursing home can vary across weeks.

- A np-chart
- B p-chart
- C c-chart
- D u-chart

Q4 Which of the following X-bar charts is most likely to depict a stable process?



- A Control chart A
- B Control chart B
- C Control chart C
- D Control chart D

Q5 If you want the false alarm rate to be 1 in 1000, i.e. 0,1%, what is the number of sigmas to be used for the control limits?

- A 6 - sigma limits
- B 3.3 - sigma limits
- C 3 - sigma limits
- D 2.5 - sigma limits

- Q6** In a semiconductor company, statistical process control is being used to monitor the thickness of the metal used in gold plating. The process has been stable with a process mean of 985 μm and a standard deviation of 5 μm (standard deviation of the individual units). Customer specifications are $1000 \pm 30 \mu\text{m}$. What is the percentage of defective units in this case?
- A It is necessary to know the number of sigmas used for the calculation of the limits of the control chart used
 - B It is necessary to know the sample size used for plotting the control chart to be able to answer the question
 - C 3,4 dpmo (defects per million opportunities)
 - D 0,13%

- Q7** Assume that an Xbar-chart is being used to monitor a quality characteristic. By increasing the sample size you can expect:
- A an increase in the fraction non-conforming
 - B an increase in the false alarm rate
 - C an increase in the power of the control chart
 - D an increase in the ARL1

- Q8** Consider the case of taking samples of shafts regularly from a process with the purpose of monitoring their length by means of a control chart. When deciding on whether to use ranges (R) or standard deviations (s) you should keep in mind that:
- A R cannot be used as a measure of dispersion (or spread)
 - B R provides a less precision estimation of dispersion (or spread) than the s
 - C As sample size increases, the use of R becomes more preferable to the use of s
 - D By-hand calculation of R tends to be more time-consuming than that of s

- Q9** Consider the case of ten psychologists who diagnose patients (from #1 to #10). The diagnosis can be either "correct" or "incorrect". The adjoining table shows the annual number of diagnosed patients per psychologist (n), the number of incorrect diagnoses (n_def) and the percentage of incorrect diagnoses (p_def). Assume that patient diagnoses are independent of each other and the psychologists' risk of incorrect diagnosis is constant. By using a control chart with **2-sigma** limits, it is possible to conclude that:
- A Only psychologist #2 should be considered a good performer
 - B Psychologists #1 and #2 should be considered good performers
 - C There is at least one psychologist who should be considered a poor performer
 - D The differences in p_def are only due to randomness

	n	n_def	p_def
#1	100	5	5,0%
#6	100	15	15,0%
#8	100	11	11,0%
#4	200	22	11,0%
#9	200	18	9,0%
#10	200	24	12,0%
#5	200	18	9,0%
#2	300	18	6,0%
#3	300	36	12,0%
#7	300	33	11,0%
Sum	2000	200	

- Q10** The staff in a hospital is monitoring the cycle time of blood tests by means of an X-MR chart with 3-sigma control limits. The adjoining table shows the cycle time of ten consecutive blood tests in hours (from #1 to #10). The average cycle time of the ten blood tests shown in the table is 67,0 hours. What can you say about the stability of the process by examining only the X-chart?

#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
44	71	91	54	67	61	83	23	68	108

- A The process seems to be stable as there are no points outside the control limits
- B The process may not be stable as there is at least one point below the LCL and none above the UCL
- C The process may not be stable as there is at least one point above the UCL and none below the LCL
- D The process may not be stable as there is at least one point below the LCL and at least one above the UCL

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Template A	ANSWER SHEET DoE	Max: 10p - Min: 0p
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The instructions for answering the DoE questions are similar to those for answering the SPC questions.

Q1 If there are 16 runs available, what is the maximum number of two-level main factors that can be studied without confounding effects? Assume that no replications are to be taken.

A	3 main factors
B	4 main factors
C	15 main factors
D	16 main factors

Note: Do not forget to fill in the exam code on this sheet!

Q2 What is the main purpose of using randomization in design of experiment?

A	To reduce the cost of the experiment
B	To reduce the noise in the experiment
C	To spread the effect of the noise more evenly throughout the experiment
D	To estimate the noise in the experiment assuming it is constant through the experiment

Q3 A fractional factorial design (see notation to the right) was conducted and in total 16 observations were collected. What is the number of degrees of freedom in this case? Assume that the number of replications is constant for all runs (or treatments).

2^{4-1}_{IV}

A	3 degrees of freedom
B	8 degrees of freedom
C	16 degrees of freedom
D	32 degrees of freedom

Q4 Continuation of the case presented in Q3. What can you conclude about the confounding pattern?

A	Confounding does not occur in this experiment
B	The effect of some main factors is confounded with the effect of other main factors
C	The effect of some main factors is confounded with the effect of two-factor interactions
D	The effect of some two-factor interactions is confounded with the effect of other two-factor interactions

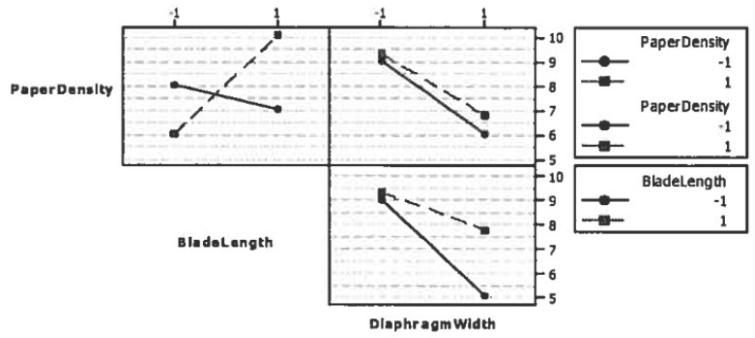
Q5 An experiment was conducted to investigate the effect of paper density, blade length and diaphragm width on the flying time of paper helicopters. The design used was a full factorial design with two observations per run. The average flying time for each run is shown in the attached cube plot. How much is the estimated effect of blade length?

A	1,6
B	3,1
C	6,2
D	It cannot be estimated as only average flying times are provided

Hand-in this sheet!

More questions on the other side!

Q6 Continuation of the case presented in Q5. The interaction plot of all two-factor interactions are shown in the attached picture. Which interaction factors has the largest effect?



- A Paper density x Blade length
- B Paper density x Diaphragm width
- C Blade length x Diaphragm width
- D Provided information is insufficient

Q7 In an experiment the two-level main factors A, B and C were tested. Only factors BC and C were found active. More information about the magnitude of the effects is provided below. How should the main effects be set in order to **maximise** the outcome?

$$|l_C| > |l_{BC}| \text{ and } |l_C| > 0 ; |l_{BC}| < 0$$

- A A+, B+ and C+
- B C+ , levels of A and B do not matter for the outcome under study
- C B-, C+ and level of A does not matter for the outcome under study
- D B+, C+ and level of A does not matter for the outcome under study

Q8 Under which circumstances should one-factor-at-a-time be used instead of design of experiments?

- A When you don't expect to find active interactions
- B When you want to save resources
- C When you want to estimate the effects of experimental factors more precisely
- D Under no circumstances

Q9 Suppose that a full factorial design with four two-level main factors A, B, C and D was conducted and no replicates were taken. The estimated effects of the three-factor and four-factor interactions are shown below. The average of these five effects is 0,8. Calculate a 95%-confidence interval of the non-active factors effects using the t-distribution.

$$l_{ABC} = 7; l_{ABD} = 10; l_{ACD} = 4; l_{BCD} = -6; l_{ABCD} = -11$$

- A $0 \pm 20,6$
- B $0 \pm 24,9$
- C $0,8 \pm 17,6$
- D $0,8 \pm 23,1$

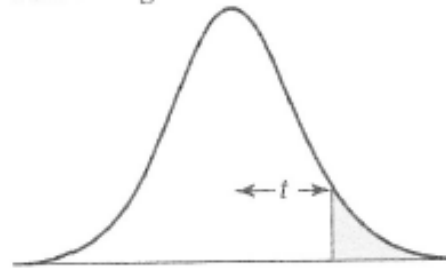
Q10 A DoE study of how the two-level main factors X1 and X2 affect the outcome Y produced the data shown in the table. For each run the average outcome and sample variance are shown as well. The average of all ten observations was 100,3 and the pooled variance 10,1. In this specific case the variance of the effects is equal 5/12 the pooled variance. Calculate a 95%-confidence interval of the non-active factors effects using the t-distribution.

- A $0 \pm 4,0$
- B $0 \pm 5,0$
- C $0 \pm 10,3$
- D $100,3 \pm 10,3$

X1	X2	Y1	Y2	Y3	Ybar	s2
Low	Low	102	98	103	101,0	7,0
High	Low	92	97	90	93,0	13,0
Low	High	102	106		104,0	8,0
High	High	104	109		106,5	12,5

Hand-in this sheet!

Note: Do not forget to fill in the exam code on this sheet!

Table B1. Probability Points of the t Distribution with ν Degrees of Freedom

ν	Tail Area Probability									
	0.4	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

n	d_2	d_3	C_4
2	1.128	0.8525	0.7979
3	1.693	0.8884	0.8862
4	2.059	0.8798	0.9213
5	2.326	0.8798	0.9400
6	2.534	0.8480	0.9515
7	2.704	0.8332	0.9594
8	2.847	0.8198	0.9650
9	2.970	0.8078	0.9693
10	3.078	0.7971	0.9727
11	3.173	0.7873	0.9754
12	3.258	0.7785	0.9776
13	3.336	0.7704	0.9794
14	3.407	0.7630	0.9810
15	3.472	0.7562	0.9823
16	3.532	0.7499	0.9835
17	3.588	0.7441	0.9845
18	3.640	0.7386	0.9854
19	3.689	0.7335	0.9862
20	3.735	0.7287	0.9869
21	3.778	0.7272	0.9876
22	3.819	0.7199	0.9882
23	3.858	0.1759	0.9887
24	3.895	0.7121	0.9892
25	3.931	0.7084	0.9896

(Control charts constants)

Probability mass function for the Poisson distribution:

$$f(k; \lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Probability mass function for the Binomial distribution:

$$f(k; n, p) = \Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

3-sigma control limits for the c-chart: $\bar{c} \pm 3\sqrt{\bar{c}}$

3-sigma control limits for the p-chart: $\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

3-sigma control limits for the Xbar-chart:

$$\mu \pm 3 \frac{\sigma}{\sqrt{n}} ; \bar{X} \pm 3 \frac{\bar{R}}{d_2 \sqrt{n}} ; \bar{X} \pm 3 \frac{\bar{s}}{c_4 \sqrt{n}}$$

3-sigma control limits for the X-chart: $\mu \pm 3\sigma ; \bar{X} \pm 3 \frac{\overline{MR}}{d_2}$

3-sigma control limits for the R-chart (also applicable for MR-chart):

$$d_2 \sigma \pm 3d_3 \sigma$$

3-sigma control limits for the s-chart: $c_4 \sigma \pm 3\sqrt{1 - c_4^2} \sigma$

Variance of the effects can be estimated as: $\sigma_{effect}^2 = \frac{4s_{pooled}^2}{r \cdot n} = \frac{4\bar{s}^2}{r \cdot n}$,

where r is the number of runs and n is the number of replications. The formula given applies when the number of replications is the same for each run, besides the assumptions of independence and homoscedasticity.

Appendix 2: Normal Probability Paper

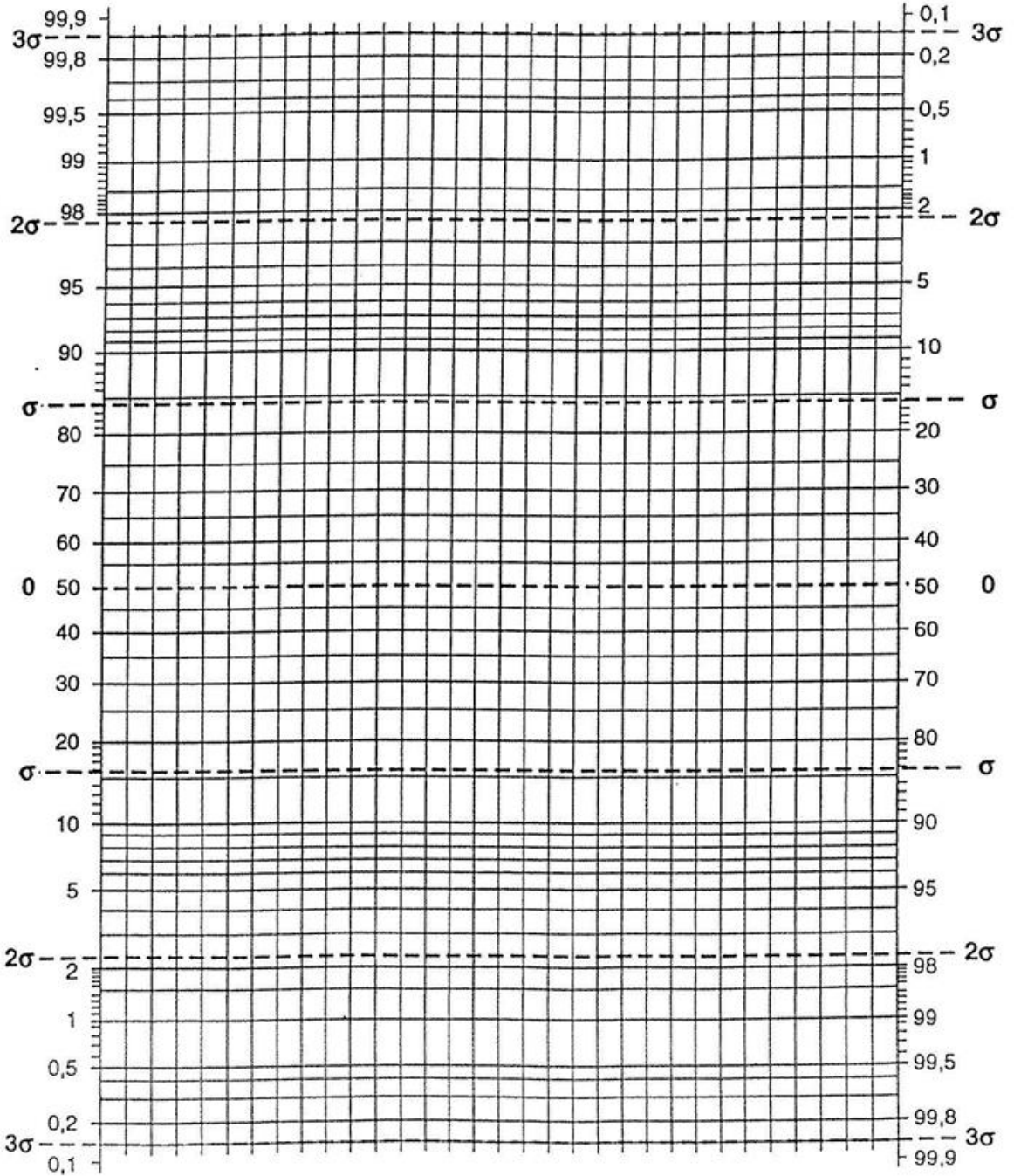
$\hat{p}_{UTG} =$ _____ %

TITEL: _____

$\hat{p}_{OTG} =$ _____ %

$\hat{\mu} =$ _____ $\hat{\sigma} =$ _____

$\hat{p}_{TOT} =$ _____ %



Q7 $\uparrow n \Rightarrow \alpha, ARL_0 \sim$
 $\downarrow \beta, ARL_1 \downarrow$
 power \uparrow

Q1 Answer: Inability to predict future performance

Q2 $\sigma_A = \sigma_B$ Tolerances_A = Tolerances_B
 $\mu_A \neq \mu_B$ $C_p = \frac{UTL - LTL}{6\sigma}$

Answer: $C_{p,A} = C_{p,B}$

$ARL_1 = \frac{1}{\text{power}} = \frac{1}{1-\beta}$
 Answer: Increased power

Q8 Answer: R less precise than S

Q3 Nr. of falls per week a Poisson
 Varying area of opportunity
 Answer: u-chart

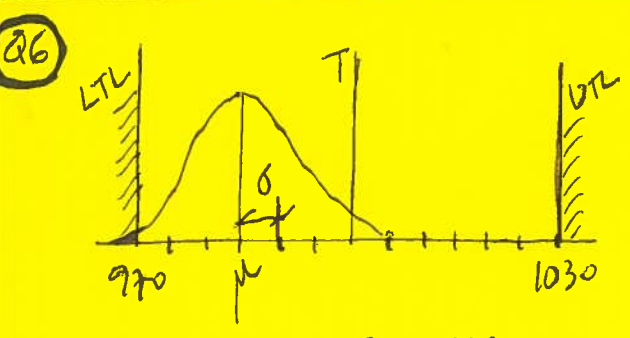
Q9 $p = \frac{200}{2000} = 10\%$

$n = 100$ $10\% \pm 2 \times \sqrt{\frac{10\% \times 90\%}{100}}$ $\begin{cases} 41.0\% \\ 16.0\% \end{cases}$

Q4 Answer: chart B

Q5 For 3-sigma control limits $ARL_0 \approx 370$
 For 6-sigma control limits $ARL_0 \approx 1/2$ billion
 Answer: 3.3-control limits

$n = 200$ $10\% \pm 2 \times \sqrt{\frac{10\% \times 90\%}{200}}$ $\begin{cases} 5.8\% \\ 14.2\% \end{cases}$



$LTL = 970, z_{LTL} = \frac{970 - 985}{5} = -3$
 $UTL = 1030, z_{UTL} = 9$
 $1 - \phi(z=3) \approx 1$ in $370 \times 2 = \frac{1}{740} \approx 0.135\%$
 $1 - \phi(z=9) \approx 0\%$
 $FNC = 0.135\% + 0\% \approx 0.13\%$

Answer: $FNC \approx 0.13\%$

$n = 300$ $10\% \pm 2 \times \sqrt{\frac{10\% \times 90\%}{300}}$ $\begin{cases} 6.5\% \\ 13.5\% \end{cases}$

Answer: Only #2 is outlier

Q10 10 observations
 9 MR-values:
 MR-values: 27; 20; 37; 13; 6; 22; 60; 45; 4
 $\bar{MR} = \frac{270}{9} = 30.0$
 $\bar{X} \pm 3 \frac{\bar{R}}{d_2 \sqrt{n}}$ ($\bar{X} \pm 3 \frac{\bar{R}}{d_2}$ $d_2(n=2) = 1.128$)
 $\bar{X} \pm \frac{3}{1.128} \bar{MR} = \bar{X} \pm 2.66 \bar{MR} =$
 $67.0 \pm 2.66 \times 30.0$ $\begin{cases} 0 \text{ hours} \\ 147 \text{ hours} \end{cases}$

Answer: no outliers

Q1 $2^4 = 16$ Answer: 4 main factors

Q2 Answer: Spread - effect of noise more evenly

Q3 total 16 observations

8 runs

	A	B	C	AB	AC	BC	ABC ^D	y ₁	y ₂
#1								x	x
⋮								⋮	⋮
#8								x	x

2 observations per run
 1 degree of freedom per run
 Answer: 8 degrees of freedom

Q9 $\sigma^2 = \frac{\sum (x - \mu)^2}{n}$
 Variance of effects of 7, 10, 4, -6, -11
 Variance of effects = $\frac{7^2 + 10^2 + 4^2 + 6^2 + 11^2}{5}$
 Variance of effects = 64,4
 Stdev of effects = $\sqrt{64,4} = 8,025$
 $t_{2,5\%, DF=5} = 2,571$

CI = $0 \pm 2,571 \times 8,025 = 0 \pm 20,6$

Answer: $0 \pm 20,6$

Q4 Answer: 2nd order confounded with 2nd order

Q5 Blade length
 High: 7,4; 6,7; 8,9; 11,35 $\bar{y}_{high} = 8,6$
 Low: 10,7; 5,4; 4,7; 7,35 $\bar{y}_{low} = 7,0$
 Effect = $8,6 - 7,0 = 1,6$

Answer: Effect of blade length = 1,6

Q10 CI = $0 \pm t_{2,5\%, DF} \times \sigma_{Effects}$
 DF = 6 $t_{2,5\%, DF=6} = 2,447$
 $\sigma_{Effects} = \sqrt{5/12 \times 10,1} = \sqrt{4,2} \approx 2,0$

CI = $0 \pm 5,0$
 Answer: $0 \pm 5,0$

Q6 Answer: Paper density x blade length

Note! Concurrent line segments indicate strong interaction
 Parallel line segments indicate weak interaction

Q7 Ex. $l_c = 10 \Rightarrow C^+$
 $l_{BC} = -5 \Rightarrow$ keep BC low \Rightarrow B and C with different signs $\Rightarrow B^-$

Answer: C^+, B^- , A should be set according to other criteria

Note!
 $S^2_{pooled} = (2S_1^2 + 2S_2^2 + S_3^2 + S_4^2) / 6$

Q8 Answer: No circumstances

Effect = $\frac{1}{2} (\pm \bar{y}_1 \pm \bar{y}_2 \pm \bar{y}_3 \pm \bar{y}_4)$
 $\sigma^2_{Effect} = (\frac{1}{2})^2 (\sigma_{y_1}^2 + \sigma_{y_2}^2 + \sigma_{y_3}^2 + \sigma_{y_4}^2) = (\frac{1}{2})^2 \sigma_y^2 (\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_4})$
 $\sigma^2_{Effect} \approx \frac{1}{4} \times S^2_{pooled} \times (\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}) = \frac{1}{4} \times \frac{5}{6} S^2_{pooled} = \frac{5}{24} S^2_{pooled}$