Quantum Informatics FKA172

Time: Monday, March 14, 2011, 08:30-12:30
Examiner: Thilo Bauch (tel 772 3397)
Allowed reference: A hand written A4 sheet (both sides may be filled) and a Chalmers approved calculator.
Grading: The points awarded for each question is indicated by that question and in case of subquestions the division of the point is also indicated. The total number of points is 24.

1. **Bloch sphere manipulation** (2 pts.)

   Given two fields $B_z(t)$ and $B_y(t)$ that couple to a qubit, describe how to take the state $|0\rangle$ to the state $(|0\rangle - i|1\rangle)\sqrt{2}$. During how long time should the fields be applied?

2. **Density matrix** (2 pts.)

   A qubit has $\langle\sigma_z\rangle = 0.8$, $\langle\sigma_y\rangle = 0$, and $\langle\sigma_x\rangle = 0.6$. Evaluate the density matrix of the qubit. Is the qubit in a mixed or pure state after a $\pi/4$-rotation about the x-axis?

3. **Rabi Hamiltonian and Jaynes-Cummings Hamiltonian** (6 pts.)

   a) The Rabi Hamiltonian of an atom in presence of an external electromagnetic field is given by

   $$\hat{H} = \frac{\hat{\mathbf{P}}^2}{2m} + V(r) + e\mathbf{r}\cdot\mathbf{E}(t) = \hat{H}_0 - \hat{\mathbf{d}}\cdot\mathbf{E}(t),$$

   where $\hat{\mathbf{d}} = -e\mathbf{r}$ is the dipole operator. Under which condition is this equation valid? (0.5)

   b) The Jaynes-Cummings Hamiltonian can be derived from the Rabi Hamiltonian by replacing the classical field $\mathbf{E}(t)$ of a single mode in a one dimensional cavity (resonator) by an operator $\hat{\mathbf{E}}(t) = \hat{E}_x(z,t) = \mathcal{E}_0(\hat{a}+\hat{a}^\dagger)\sin(kz)$, resulting in:

   $$\hat{H} = -\frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega\hat{a}^\dagger\hat{a} + g(\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-). \tag{1}$$

   This Hamiltonian acts in the Hilbert space spanned by the vectors $|g\rangle|n\rangle$ and $|e\rangle|n\rangle$, where $|g\rangle$ and $|e\rangle$ are the ground state and excited state of the two level system, respectively. $|n\rangle$ are the eigenstates of the photon number operator $\hat{n}$.

   - i) What is the meaning of the three terms in the hamiltonian? (1 pt.)
   - ii) What approximation was used to derive this hamiltonian and what is the physical meaning of this approximation? (0.5 pt.)

- iii) From the above Hamiltonian (Equation 1) derive the Hamiltonian in the two dimensional subspace spanned by $|g\rangle|n+1\rangle$ and $|e\rangle|n\rangle$. (Hint: Evaluate the matrix elements in the two dimensional subspace and represent the final result using the Pauli matrices.) (1 pt.)
- iv) Depict the energy spectrum of the uncoupled ($g = 0$) and coupled ($g \neq 0$) atom-photon states for zero detuning $\omega = \omega_0$. What is the quantum electrodynamic Rabi frequency? (1 pt.)

c) For large detuning between the field frequency and atom transition frequency ($\hbar|\omega - \omega_0| \gg g$) we obtain the dispersive Hamiltonian:

$$\hat{H}_{\text{disp}} = -\frac{1}{2}\left(\hbar\omega_0 + \frac{g^2}{\hbar\Delta}\right)\sigma_z + \left(\hbar\omega - \frac{g^2}{\hbar\Delta}\sigma_z\right)\hat{a}^\dagger\hat{a}.$$

- i) What is the frequency of the cavity when the atom (qubit) is in the excited state (0.5 pt.)
- ii) How would you readout the (atom) qubit state, and would there be any back action on the qubit during readout? (1.5 pts.)

4. **Bloch equations** (3 pts.)

   The Bloch equations describe the motion of a spin-1/2 on/in the Bloch sphere $\mathbf{r(t)} = (r_x(t), r_y(t), r_z(t))$ under the influence of control fields $\mathbf{B}(t)$ and dissipation described by phenomenological timescales for relaxation/mixing T_1 and dephasing T_2

   $$\dot{\mathbf{r}} = -\frac{1}{\hbar}\mathbf{B}\times\mathbf{r} - \frac{1}{T_1}\left(r_z - r_z^0\right)\hat{z} - \frac{1}{T_2}\left(r_x\hat{x} + r_y\hat{y}\right), \tag{2}$$

   a) How are the two timescales T_1 and T_2 related to the noise fields $\delta\mathbf{B}_n = (\delta B_{nx}, \delta B_{ny}, \delta B_{nz})$ and their spectral densities? (2 pts.)
   b) What is the meaning of r_z^0 and what is its value for the following two cases: zero temperature and infinite temperature? (1 pt.)

5. **Superconducting qubits** (1 pt.)

   a) What are the pros and cons of using superconducting qubits for the realization of a quantum computer? (0.5 pt.)

   b) A typical phase qubit has at the working point ($0.9 < I_b/I_c < 1$) a transition frequency between the ground state and the first excited state $\nu_{01} = \omega_{01}/2\pi \simeq 5$GHz. What is the temperature range at which you can operate the qubit? (Boltzmann constant $k_B \simeq 1.38 \cdot 10^{-23}$ J/K, Planck constant $h \simeq 6.63 \cdot 10^{-34}$ Js) (0.5 pt.)

6. **Measurements of Rabi oscillations in a phase qubit** (3 pts.)

   a) Explain how the measurement of Rabi oscillations between the ground state and the first excited state are performed in a phase qubit (current biased Josephson junction). Include sketches of the timing sequence for the bias current and the harmonic microwave signal inducing the Rabi oscillations. Here we assume that the frequency of the harmonic microwave signal f_{mw} is equal to the transition frequency of the qubit ν_{01}.(1 pt.)

b) Add to the timing sequence sketches of the tilted washboard potential, including energy levels and their population, during

- the initialization (working point) of the qubit
- the microwave manipulation of the qubit
- the read out of the qubit (1 pt.)

c) During the read out of your qubit you want to discriminate between the qubit being in the ground state or in the first excited state. In order to discriminate between the two states how would you choose the amplitude of the read out current pulse? Include a sketch showing the switching probability from the zero voltage state to the finite voltage state of the Josephson junction as function of read out current pulse amplitude for both the qubit being in the ground state and the qubit being in the first excited state. (1 pt.)

7. **The CNOT gate** (1 pt.)

   For the CNOT gate write down
   a) the circuit symbol
   b) the truth table, i.e. the output states for the different input basis states
   c) the corresponding unitary matrix

8. **Quantum Teleportation** (2 pts.)

   Explain the concept of quantum teleportation, including the initial setup, the people involved and their resources, the main steps and what is actually teleported.

9. **The No-Cloning Theorem** (1 pt.)

   What is stated in the no-cloning theorem and how is it related to teleportation?

10. **Deutsch's Algorithm** (3 pts.)

    In Fig.1 a circuit implementing Deutsch's algorithm is shown. What is the problem solved by this algorithm? How much faster is this algorithm compared to any classical algorithm for the same problem? Show explicitly how the algorithm works, i.e. write down the states $|\Psi_0\rangle$, $|\Psi_1\rangle$, $|\Psi_2\rangle$, $|\Psi_3\rangle$, and specify what measurement should be performed and how the result answers the problem.

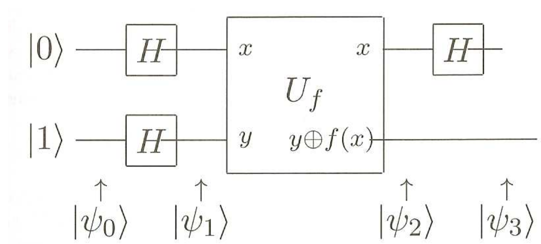

Figure 1: *A circuit implementing Deutsch's algorithm*