Chalmers University of Technology, Department of Applied Mechanics
Teacher: Mattias Wahde, Tel. 772 3727

Exam in FFR 105 (Stochastic optimization algorithms), 2008-10-22, 14.00-18.00, V.

It is allowed to use a calculator, as long as it cannot store any text. Furthermore, mathematical tables (such as Beta, Standard Math etc.) are allowed, provided that no notes have been added. However, it is *not* allowed to use the course book during the exam. Note! In all problems involving analytical calculations, derivations, proofs etc., show *clearly* how you arrived at your answer, i.e. include intermediate steps etc.

There are 4 problems in the exam, and the maximum number of points is 25.

1. (a) Many operators and concepts (and different versions thereof) have been defined in connection with evolutionary algorithms (EAs). Describe, in detail, the following concepts:

    i. Elitism (1p)
    ii. Fitness ranking (1p)
    iii. Creep mutations (real-number creep) (1p)

    You should *not* write Matlab code, but make sure to describe the three concepts in such a way that it would be possible to write Matlab code, based on your description.

   (b) Roulette-wheel and tournament selection are commonly used methods for selection in EAs. Consider a case where a single individual is to be selected from a population in which the fitness values are $F_1 = 1, F_2 = 4, F_3 = 9, F_4 = 16$, $F_5 = 25$, using either (i) roulette-wheel selection or (ii) tournament selection with a tournament size of two, and with tournament selection probability $p_{\text{tour}} = 0.75$. What is the probability of selecting individual 4 (with fitness = 16) using

    i. Roulette-wheel selection (1p)
    ii. Tournament selection (1p)

   (c) Convexity (of the objective function) is a desirable property in optimization problems. Formally, if $\mathbf{S} \in \mathbf{R}^n$ is a convex set and $f(\mathbf{x})$ is a convex function defined on $\mathbf{S}$, then any local minimum is also a global minimum. *Prove* this result, using the properties of convex functions. Note: Make sure to use clear statements and formulations, such that the proof can be followed, in detail, from the first step to the last. (2p)

   (d) Is the function

    $$f(x_1, x_2) = 4x_1^2 + 2x_2^2 - 3x_1x_2 \tag{1}$$

    convex or not? Motivate your answer clearly! (1p)

   (e) In stochastic optimization algorithms, such as EAs, ant colony optimization (ACO) and particle swarm optimization (PSO), there is always a tradeoff between *exploration* and *exploitation* of the results already found. Describe, in detail, how this tradeoff is managed *in the case of PSO*. In your description, include any equations that may be useful. (2p)

2. Determine (analytically, using one or several of the classical optimization methods covered in the course) the minimum value taken by the function

$$f(x_1, x_2) = 2x_1^2 - 4x_1 + x_2^2 + 2x_2, \tag{2}$$

over the set

$$\mathbf{S} = \{(x_1, x_2) \colon 2x_1^2 + x_2^2 \leq 12\}. \tag{3}$$

Make sure to describe all steps in the calculation clearly. (4p)

3. Ant colony optimization (ACO), which is inspired by the behavior of ants, is typically used for solving routing problems, such as the traveling salesman problem (TSP). Several ACO algorithms have been defined.

   (a) Describe the algorithm Ant system (AS) in detail. Make sure to provide a clear list of the various steps in the algorithms, as well as a brief explanation of each step. You *should not* write Matlab code, but your presentation of the algorithm should be sufficiently clear to make an implementation possible, based on your description. You may use the TSP as a specific example in the description. (3p)

   (b) Max-min ant system (MMAS) is another version of ACO, derived from AS. List and describe clearly the *differences* between MMAS and AS. (2p)

   (c) In MMAS explicit lower and upper bounds are introduced on the pheromone levels. However, the explicit *upper* bound is, in fact, unnecessary. Prove rigorously (for MMAS) that the maximum pheromone level on any edge e_{ij} cannot exceed f^*/ρ, where f^* is the value of the objective function for the optimal solution (i.e. $1/D^*$ in the case of TSP, where D^* is the length of the shortest possible path) and $\rho \in]0, 1]$ is the evaporation rate. (2p)

4. In analytical studies of EAs, it is common to use the Onemax problem, for which the value of the fitness function for a given (binary) chromosome equals the number of 1s in the chromosome. For this simple problem, one can derive an expression for the expected runtime (number of evaluations) for an EA with a single individual, which is modified using mutations only. In this EA, a mutated individual is kept if and only if it is better (i.e. its chromosome contains more 1s) than the previous individual.

   (a) Consider a chromosome of length m with l 0s (and, therefore $m-l$ 1s). Let the mutation rate be p_{mut}. Derive an approximate expression for the probability of improving this chromosome (i.e. increasing the number of 1s). The expression should summarize a case in which none of the 1s mutate, and at least one of the 0s does. (1p)

   (b) Using the probability estimate derived in (a), derive an expression for the expected number of evaluations needed to reach a chromosome consisting only of 1s, starting from a chromosome with $\frac{m}{2}$ 0s. Let the mutation rate be equal to k/m, for some value of $k \ll m$. (3p)