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Exam in FFR105/FIM711 (Stochastic optimization algorithms), 2022-10-26, 14.00-18.00

The examiner will visit the exam room twice, around 15.00 and around 17.00. It will be possible to review your results (exam and home problems) from Nov. 17 and onwards.

In the exam, it is allowed to use an approved ("typgodkänd") calculator. No other calculators are allowed. Furthermore, it is *not* allowed to use the course book, lecture notes, or slides from the course, during the exam. It is allowed to use mathematical tables (e.g. Beta, Standard Math etc.), as long as no notes have been added.

Note! For Problem 1 below, you should *only* give the answers, i.e., a single letter (A), (B), (C), (D), or (E) for each question, all collected on a single page. For Problems 2 – 5 you should provide a clear description of how you arrived at your answer, including all relevant intermediate steps. *Only giving the answer will not give any points for Problems 2 – 5.*

There are five problems in the exam, and the maximum number of points is 25.

1) This problem consists of ten questions, each of which is associated with five alternative answers, of which one is correct. For each of the 10 questions below, you should therefore *only give one answer*, in the form of a letter: (A), (B), (C), (D), or (E). Answer the questions in order (1.1, 1.2, …,1.10) and mark your answers *clearly* (e.g., "1.1: A"). For this problem, the number of points awarded is determined as follows: 10 correct answers: 10p, 9 correct answers: 8p, 8 correct answers: 6p, 7 correct answers: 5p, 6 correct answers: 4p, 5 correct answers: 3p, 4 correct answers: 2p, 3 correct answers: 1p, 2 or fewer correct answers: 0p.

1.1. When using PSO, it is essential to keep the swarm coherent. Which of the following statements is true?

A. Both velocities and positions should be restricted.
B. Only positions should be restricted.
C. Positions should be restricted if particles venture outside the initial range.
D. Only velocities should be restricted.
E. Positions should be restricted if the inertia is smaller than 1.

1.2. Consider the function $f(x_1, x_2) = 4x_1^2 + 2x_2^2 - x_1x_2$ and let H denote the Hessian matrix. Which of the following statements is true?

A. The eigenvalues of H are both negative, so f is convex.
B. The eigenvalues of H are both positive, so f is convex.
C. One eigenvalue of H is positive, and one is negative, so f is not convex.
D. The eigenvalues of H are both negative, so f is not convex.
E. The eigenvalues of H are both positive, so f is not convex.

1.3. Consider a genetic algorithm in which individuals are selected using tournament selection, with a given value of p_{tour} and with a tournament size of three. In a single selection step (selecting one individual from the tournament with three individuals), what is the probability (p) of selecting the second-best individual.

A. $p = p_{\text{tour}}^2$
B. $p = (1 - p_{\text{tour}})p_{\text{tour}}$
C. $p = p_{\text{tour}}^3$
D. $p = (1 - p_{\text{tour}})p_{\text{tour}}^2$
E. $p = (1 - p_{\text{tour}})^2$

1.4. Consider a case in which the Lagrange multiplier method is applied to the problem of finding the minima of a continuously differentiable function $f(x_1, x_2)$ subject to a continuously differentiable equality constraint $h(x_1, x_2) = 0$, both defined in a bounded region (i.e., such that it can be contained in a disc of radius R, where R is finite). Which of the following statements is true? For the problem at hand, the points found by the Lagrange multiplier method …

A. … contain local minima and maxima, but not any global minima or maxima.
B. … contain local and global minima, but no maximum, neither local nor global.
C. … contain local and global maxima, but no minimum, neither local nor global.
D. … contain local and global optima, both minima and maxima.
E. … contain only global minima and maxima.

1.5. What are the stationary points of the function $f(x) = 2x^3 - 3x^2 - 12x + 4$?

A. The stationary points are x = 4 and x = 12.
B. The stationary points are x = -2 and x = 1.
C. The stationary points are x = -1 and x = 2.
D. The stationary points are x = 3 and x = 6.
E. The stationary points are x = -1 and x = 1.

1.6. Consider a genetic algorithm with a population of five individuals, with fitness values $F_1 = 1$, $F_2 = 3$, $F_3 = 4$, $F_4 = 6$, and $F_5 = 10$. Assuming that tournament selection is used, with $p_{\text{tour}} = 0.75$, what is the probability (in a single selection step) of selecting individual 2 (whose fitness is equal to 3)?

A. 0.12
B. 0.10
C. 0.18
D. 0.24
E. 0.16

1.7. Consider a binary chromosome in a GA, which has been generated via selection and crossover, and is then going to be mutated using the standard mutation procedure, with mutation rate $p_{\text{mut}} = 1/m$, where m is the chromosome length. Which of the following statements is true?

A. The number of genes that undergo mutation will range from 0 to m
B. The number of genes that undergo mutations will range from 0 to m/2
C. Either 0 or 1 gene will undergo mutation
D. Either 0, 1, or 2 genes will undergo mutation
E. Exactly 1 gene will undergo mutation

1.8. The Ant system (AS) algorithm can be applied in many different problems. Let τ_{ij} denote the pheromone matrix and η_{ij} the visibility matrix. Which of the following statements is true: For any problem where AS is applied, at the end of a run lasting a given (finite) number of iterations …

A. … the pheromone matrix is symmetric, whereas the visibility matrix is not.
B. … the visibility matrix is symmetric, whereas the pheromone matrix is not.
C. … the symmetry (or lack thereof) of either matrix depends on the problem.
D. … both the visibility matrix and the pheromone matrix are symmetric.
E. … neither the visibility matrix nor the pheromone matrix is symmetric.

1.9. Consider a standard genetic algorithm (GA) where either tournament selection (TS) or (standard) roulette-wheel selection (RWS), without fitness ranking, is used, and where the fitness values (f) are in the range [0,1]. Which of the following statements is true? An individual whose fitness is equal to 0 …

A. … can be selected with TS but not with RWS.
B. … can be selected with RWS but not with TS.
C. … cannot be selected with either method.
D. … can be selected with both methods.
E. … can be selected with both methods, but only in the first generation.

1.10. Consider a case in which ant system (AS) is being used for solving the travelling salesperson problem (TSP), and where three nodes remain to be selected to complete the path of a given ant. From the current node, the visibilities of those nodes are 1, 2, and 5, respectively. Moreover, the parameters α and β are both equal to 1, and the pheromone level (τ) is equal to 1 on all edges. What is the probability of selecting the third node (with visibility 5) as the next node?

A. 0
B. 2/17
C. 3/8
D. 5/8
E. 1

2) Using the Lagrange multiplier method, find the minimum value and the maximum value of the function

$$f(x, y, z) = x^2 + y^2 + z^2$$

subject to the constraint

$$x^4 + y^4 + z^4 = 1.$$

List also the locations (x, y, z) of *all* points where the function takes either the minimum value or the maximum value. When solving this problem, make sure to include all relevant intermediate steps, and to carefully motivate the steps in the calculations. (4p)

1	2	1	6	3	3	2	2	1	2	1	4	1	3	2	3	1	1	1	5	1	2	1	3

Figure 1: The chromosome for Problem 3.

3) Consider the LGP chromosome shown in Figure 1. As usual in LGP, the chromosome defines a list of instructions, each consisting of four genes. The first gene defines the operator from the set $O = \{+, -, *, /\}$ (i.e., the four standard arithmetic operators), such that $1 \Leftrightarrow +, 2 \Leftrightarrow -$, and so on. The second gene defines the destination register, from the set $R = \{r_1, r_2, r_3\}$. The third and fourth genes define the operands, from the set $A = \{r_1, r_2, r_3, c_1, c_2, c_3\}$, where the constant registers take the values $c_1 = 1, c_2 = 2$, and $c_3 = -1$. When evaluating the chromosome for a general variable x, the variable registers are initialized as follows: $r_1 = x, r_2 = r_3 = 0$, and the output, namely a function $g(x)$, is taken as the content of r_2 at the end of the evaluation. What function $g(x)$ is represented by the chromosome in Fig. 1? Write your answer in the form of a polynomial, $g(x) = a_0 + a_1 x + a_2 x^2 + \cdots$ (3p)

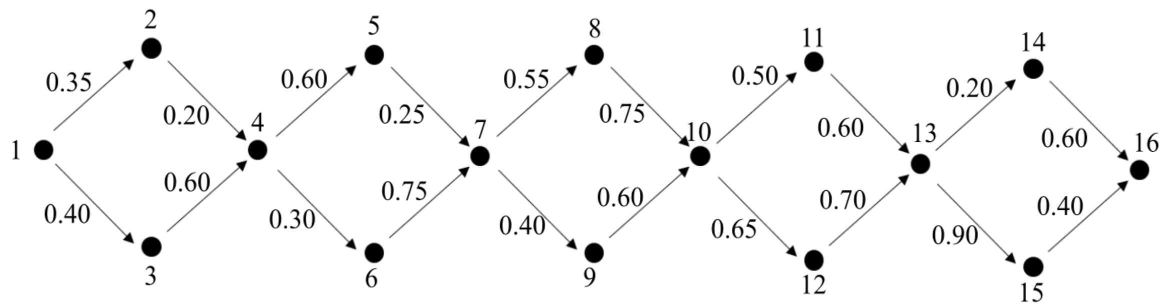

Figure 2: The chain construction graph for Problem 4. The nodes are enumerated from 1 to 16. The decimal numbers on the edges represent the pheromone levels.

4) Chain construction graphs can be used when applying the ant system (AS) algorithm in function optimization tasks. Consider a simple case where the task is to find the minimum of the function

$$f(x) = \left(x - \frac{3}{16}\right)^2$$

The variables x are obtained by artificial ants traversing the chain construction graph in Figure 2, following the standard procedure for such graphs: At the central nodes (1, 4, 7 etc.), the ant uses the standard ACO equation for $p(e_{ij}|S)$ (with $\alpha = \beta = 1$) to take either an up-move or a down-move. An up-move generates a 1 and a down-move generates a 0. Then, the ant moves deterministically to the next central node, and so on, until the last central node (16) is reached. The resulting bit string, which is denoted $a_1 a_2 \ldots$, is then converted to a decimal number in the open range [0,1[, as $x = \sum_j a_j 2^{-j}$ where the sum runs from 1 to 5. Note that the visibility is equal to 1 for all edges, in this case, and the pheromone levels are given in Figure 2.

Consider a population consisting of five ants. The population is evaluated (once) in its entirety, without any pheromone changes. What is the probability that at least one of the five ants generates the value of x that corresponds to the global minimum of $f(x)$? (4p)

5) Consider a simple, one-dimensional application of PSO, in which the goal is to minimize the function $f(x) = (x - (1/4))$^2, using a swarm size of three. Initially, the three particles are located at $x = -1/3$ (particle 1), $x = 0$ (particle 2), and $x = 3/4$ (particle 3), and their speeds are $v = 3$ (particle 1), $v = 1/4$ (particle 2), and $v = -1$ (particle 3). The parameters α and Δt are both equal to 1, w is (here) kept *constant* at the value 1, and $c_1 = c_2 = 2$. Moreover, assume (somewhat unrealistically) that the random numbers q and r are (always) both equal to 1. The initial range $[x_{\min}, x_{\max}]$ is equal to $[-2,2]$, and the particle speeds are restricted to a maximum of 4. Given these parameters, determine, under the PSO algorithm

(a) the velocities and positions of all particles after one iteration (i.e. one updating step for both velocities and positions). (2p)
(b) the velocities and positions of all particles after two iterations. (2p).
Note: For part (b), points are only given if the results from part (a) are correct.