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Exam in FFR 105 (Stochastic optimization algorithms), 2020-10-28 14.00-18.00

The exam will be monitored via Zoom. It will be possible to review your exam results during the week starting Nov. 22.

Note! In each problem, show *clearly* how you arrived at your answer, i.e. include all relevant intermediate steps. Only giving the answer will result in zero points on the problem in question.

There are four problems in the exam, and the maximum number of points is 25.

1. (a) Use Newton-Raphson's method to find the stationary point of the function

$$f(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 - x. \qquad (1)$$

Start by finding a suitable search range (i.e. one that includes the minimum), for example by plotting the function. Motivate clearly the choice of your search range. Then carry out the steps of the Newton-Raphson method until you get an answer that is accurate to five decimal places. (3p)

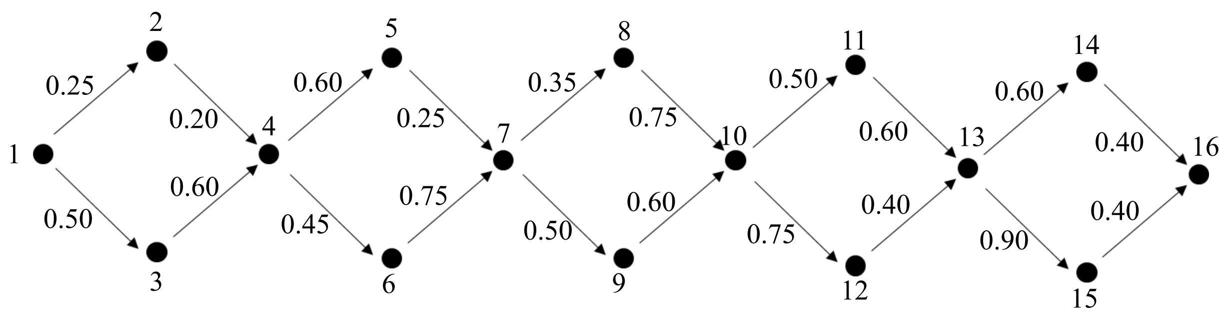
(b) Use the Lagrange multiplier method to compute the smallest distance from the point $(4, 0, 0)^{\mathrm{T}}$ to a point on the surface $x_1^2 + x_2^2 - x_3^2 = 1$. (4p)

(c) Find all global minima and maxima of the function

$$f(x_1, x_2) = \exp\left(x_1 - \frac{1}{3}x_1^3 - x_2^2\right), \qquad (2)$$

in the set S defined by $-2 \le x_1 \le 2$ and $-2 \le x_2 \le 2$. (5p)

2. Consider a population (in a genetic algorithm) with fitness values 1,3,5,6, and 30. What is the probability of selecting the best individual, in a single selection step, using (a) roulette-wheel selection and (b) tournament selection with a tournament size of 2 and a tournament selection parameter p_{tour} of 0.75? What conclusions, if any, can be drawn? (3p)

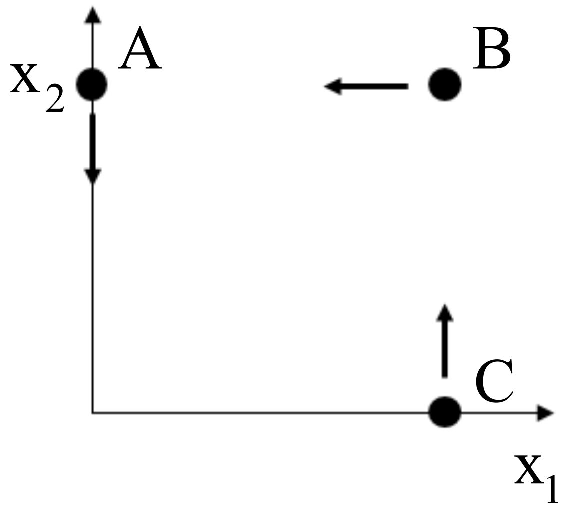


3. Chain construction graphs can be used in order to apply the ant system (AS) algorithm in function optimization tasks. Consider a simple case where the task is to find the minimum of the function

$$f(x) = \left(x - \frac{3}{4}\right)^2. \tag{3}$$

The variables x are obtained from the construction graph (see the figure above, where the nodes are enumerated from 1 to 16) as described in the course book. That is, for each step *from* the nodes *with indices* $3L+1$, $L = 0, 1, 2, \ldots$, a 0 (zero) is generated if a down-move is taken (e.g. from node 1 to node 3, that is following edge e_{31}) and a 1 (one) is generated if an up-move is taken (e.g. from node 1 to node 2). The path generation proceeds according to the standard ACO equation for $p(e_{ij}|S)$, with $\alpha = \beta = 1$. The edges e_{ij} are shown as arrows in the figure, and the number on each arrow is the corresponding pheromone level τ_{ij}. The visibility is assumed to be equal to 1 for all edges. The resulting bit string (denoted $a_1 a_2 \cdots a_n$, where n is the number of bits, and where a_1 is generated when taking the step from node 1 etc.) is then converted to a decimal number in the open range $[0, 1[$ as $x = \sum_{j=1}^{n} 2^{-j} a_j$.

Consider a population consisting of five ants. The population is evaluated (once) in its entirety, without any pheromone changes. What is the probability that at least one of the five ants generates the value of x that corresponds to the global minimum of $f(x)$? (5p)



4. Consider a situation where PSO is used for solving the problem of minimizing

$$f(x_1, x_2) = x_1^2 + 2x_2^2, \qquad (4)$$

with a swarm of three particles. At initialization, the three particles A, B, and C are located at $(0,1)^{\mathrm{T}}$, $(1,1)^{\mathrm{T}}$, $(1,0)^{\mathrm{T}}$, respectively, as shown in the figure, and the particle speeds are all equal to 1. Particle A moves in the negative x_2-direction, Particle B in the negative x_1-direction, and Particle C in the positive x_2-direction. The constants c_1 and c_2 are both set to 2 whereas α and Δt are both set to 1. The inertia weight w is assumed to be constant at 1, and the random numbers (q and r) are both assumed to be equal to 0.5 for all particles, at all times. Velocity restrictions are applied in a componentwise manner, such that all velocity components are (individually) limited to a maximum magnitude of 1. Start by finding the particle best position vector $\mathbf{x}_i^{\mathrm{pb}}$ (for each particle i, i =A,B,C) and the swarm best position vector $\mathbf{x}^{\mathrm{sb}}$. Then

(a) Compute the new velocites for all particles. (2p)

(b) Compute the new positions for all particles. (1p)

(c) Plot the new positions and velocities for all particles and again compute the particle best position vectors and the swarm best position vector. (2p)