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Exam in FFR 105 (Stochastic optimization algorithms), 2019-10-30, 14.00-18.00, M.

The examiner will visit the exam rooms twice, around 15.00 and around 17.00. It will be possible to review your results (exam and home problems) any day after Nov. 17.

In the exam, it is allowed to use a calculator, as long as it cannot store any text. Furthermore, mathematical tables (such as Beta, Standard Math etc.) are allowed, provided that no notes have been added. However, it is *not* allowed to use the course book, or any lecture notes from the course, during the exam.

Note! In problems involving computation, show *clearly* how you arrived at your answer, i.e. include intermediate steps etc. Only giving the answer will result in zero points on the problem in question. There are four problems in the exam, and the maximum number of points is 25.

1. (a) Premature convergence is a common problem in evolutionary algorithms. There are different ways of avoiding this problem, by modifying or extending the various operators used (or their parameters). Consider a genetic algorithm that (before modification) uses tournament selection (TS) and standard operators for crossover and mutation, with typical parameter settings. How can the algorithm be modified in order to prevent premature convergence (assuming that TS is used even after the modification)? Describe *two* ways for doing so. (2p)

   (b) In genetic algorithms, the concept of *genes* is central. In the biological counterpart, genes serve the purpose of providing the necessary information for generating proteins by means of a process that involves two major steps. Name and *describe* the two steps. (2p)

   (c) Tournament selection (TS) and roulette-wheel selection (RWS) are two commonly used selection operators in evolutionary algorithms.

      i. Consider a population consisting of five individuals with the fitness values $F_1 = 3$, $F_2 = 6$, $F_3 = 7$, $F_4 = 10$, and $F_5 = 12$. Using tournament selection with a tournament size of two and a tournament selection parameter of 0.8, what is the probability (in a single selection step) of selecting individual 4? (1p)

      ii. Roulette-wheel selection relies on the cumulative, normalized fitness sum, denoted ϕ_j. Write down the expression for ϕ_j, and explain clearly how it is used in RWS. (2p)

      iii. Consider again the population from part (i). If the random number $r = 0.3$ is drawn, which individual will be selected assuming that now RWS is used? (1p)

2. (a) In the gradient descent method, starting from a given point $\mathbf{x}_j$ (where $\mathbf{x}$ is a vector and the index enumerates the iterations) iterates are computed such that, once the search direction has been determined, the next iterate $\mathbf{x}_{j+1}$ depends only on the step length η, so that the *function value* at that point can be expressed as some function $\phi(\eta)$. Consider now the problem of minimizing the function $f(x_1, x_2) = 2x_1^2 + 3x_1x_2 + x_2^2 - 4$ using gradient descent, starting from the point $(x_1, x_2)^{\mathrm{T}} = (1, 1)^{\mathrm{T}}$. Find and write down the (non-normalized) search direction (i.e. a vector with two components) and the expression for the next iterate, inserting numerical values. Then derive (and simplify as much as possible) the expression for $\phi(\eta)$, again with numerical values inserted. Give your answer in the form $a_2\eta^2 + a_1\eta + a_0$, where a_0, a_1, and a_2 are constants. Note: You do *not* have to carry out the line search. It is sufficient that you find the search direction, the next iterate, and $\phi(\eta)$! (2p)

   (b) The Lagrange multiplier method is often used in problems with equality constraints. Use the Lagrange multiplier method to find the point on the sphere $x_1^2 + x_2^2 + x_3^2 = 4$ that is closest to the point $\mathbf{p} = (3, 1, -1)^{\mathrm{T}}$. (3p)

3. In ant colony optimization (ACO), a population of artificial ants cooperate to find the solution of a problem expressed in the form of a graph search. A common special case is the travelling salesman problem (TSP).

   (a) ACO is based on the cooperative behavior of ants and, in particular, a special form of communication used by ants. Name and *describe* this form of communication. (1p)

   (b) The (probabilistic) method for generating paths is a central feature of ACO in general, and ant system (AS) in particular. Write down the general equation for the probability $p(e_{ij}|S)$ for taking a step from a node j to another node i for the special case of TSP, given a path fragment S. Describe carefully all variables and parameters in the equation, and give typical numerical values for the parameters. (2p)

   (c) For TSP, AS generally finds the nearest-neighbour path (from a given start node) very quickly. Assuming that the pheromone level on all edges is equal to some value $\tau_0 > 0$, explain clearly *why* AS easily finds the nearest-neighbour path. (1p)

   (d) Consider a two-dimensional TSP problem with four nodes located at $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -2)$, and where the ant starts from the first node, i.e. at $(1, 0)$. Using AS, what is the probability that this ant will follow the nearest-neighbour path, assuming that the pheromone levels are equal to $\tau_0 > 0$ on all edges, and the parameters α and β are equal to 1 and 2, respectively? (2p)

4. (a) In particle swarm optimization (PSO) there is a specific mechanism that handles the tradeoff between exploration and exploitation. Write down the general equation for the velocity updates in PSO and describe, in detail, the mechanism just mentioned. (2p)

(b) Consider now a simple one-dimensional application of PSO, in which one is trying to minimize the function $f(x) = (x - \frac{1}{4})^2$, using a swarm size of three. Initially the three particles are located at $x = -1/3$ (particle 1), $x = 0$ (particle 2), and $x = 3/4$ (particle 3), and their speeds are $v = 3$ (particle 1), $v = 1/4$ (particle 2) and $v = -1$ (particle 3). The parameters α and Δt are both equal to 1, w is (here) kept *constant* at the value 1, and $c_1 = c_2 = 2$. Moreover, assume (somewhat unrealistically) that the random numbers q and r are always equal to 1. The initial range $[x_{\text{min}}, x_{\text{max}}]$ is equal to $[-2, 2]$, and the particle speeds are thus restricted to a maximum of 4. Given these parameters, determine, under the PSO algorithm

i. ... the velocities and positions of all particles after one iteration (i.e. one updating step for both velocities and positions). (2p)

ii. ... the velocities and positions of all particles after two iterations (2p)