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Exam in FFR 105 (Stochastic optimization algorithms), 2011-10-17, 14.00-18.00, V.

It is allowed to use a calculator, as long as it cannot store any text. Furthermore, mathematical tables (such as Beta, Standard Math etc.) are allowed, provided that no notes have been added. However, it is *not* allowed to use the course book, or any lecture notes from the course, during the exam.

Note! In problems involving computation, show *clearly* how you arrived at your answer, i.e. include intermediate steps etc. Only giving the answer will result in zero points on the problem in question.

There are four problems in the exam, and the maximum number of points is 25.

1. (a) When generating new individuals in a genetic algorithm, several different operators are used, namely selection, crossover and mutation. Name and describe two selection methods. For each method, you should include a clearly described algorithm for selecting one individual, given a random number r in the range $[0, 1[$. (3p)

   (b) Particle swarm optimization (PSO) is a stochastic optimization method somewhat similar to genetic algorithms. Write down the standard PSO algorithm and describe it in detail. Include clear definitions of all variables and parameters used in your description. In particular, describe how the tradeoff between exploration and exploitation (of the results already found) is handled in PSOs. (4p)

   (c) Newton's method is an iterative method for finding local optima of an objective function $f(\mathbf{x})$. In the one-dimensional case, the method is referred to as the Newton-Raphson method. Starting from the Taylor (series) expansion of $f(x)$, *derive* Newton-Raphson's method, i.e. the equation that determines how to obtain the new iterate x_{j+1}, starting from the previous iterate x_j. Note: Provide a clear derivation of the equation. Just writing down the equation will not give any points. (2p)

   (d) In optimization, convexity of the objective function is a desirable property. Determine whether or not the function

$$f(x_1, x_2) = 4x_1^2 - 5x_1x_2 + 3x_2^2 - 7x_2 - 4 \tag{1}$$

   is convex. (1p)

2. Consider a function adaptation task in which linear genetic programming (LGP) is used for finding an unknown function $f(x)$ based on measurements taken for several different values of x. The LGP chromosomes consist of a sequence of instructions, each represented using four genes. The first gene in each instruction represents the operator, the second gene reprents the destination register, and the two remaining genes are the operands. In this task, there are three variable registers (denoted r_1, r_2, and r_3), and three constant registers (denoted c_1, c_2, and c_3). There are four operators, namely o_1 (addition), o_2 (subtraction), o_3 (multiplication) and o_4 (division). Initially the constant registers are set as $c_1 = 1$, $c_2 = 2$ and $c_3 = -1$. The variable registers are initiated as $r_1 = x$, $r_2 = r_3 = 0$. The output (i.e. the estimate $\hat{f}(x)$) is taken as the contents of r_2. The operands are chosen from the set $\{a_1, \ldots a_6\} = \{r_1, r_2, r_3, c_1, c_2, c_3\}$.

   (a) Consider an LGP chromosome given by

   $$c_1 = 1214\,3315\,3123\,3333\,1323\,4213. \tag{2}$$

   Which function is obtained when decoding this chromosome? (2p)

   (b) During mutation, the fourth gene in the chromosome c_1 is mutated from 4 to 1. What will be the corresponding function? (1p)

3. Ant colony optimization (ACO), which is inspired by the behavior of ants, is typically used for solving routing problems, such as the traveling salesman problem (TSP).

   (a) Consider the construction graph (for TSP) shown in Fig. 1 (see next page). If the level of artificial pheromone is equal to 0.5 for all edges e_{ij}, what is the probability that an ant will follow the nearest neighbour path, starting from node 1? For the parameter values, choose $\alpha = 1$ and $\beta = 2$. Make sure to include all relevant intermediate steps in your calculations! (3p)

   (b) Several ACO algorithms have been defined, one of which is the Max-min ant system (MMAS) in which (among other things) pheromone limits are imposed. Consider again the construction graph for TSP in Fig. 1. Assuming that MMAS is being used, with $N = 4$ artificial ants, and that the initial pheromone levels τ_{ij} in this case are equal to $1/(\rho D_{\text{nn}})$ for all edges e_{ij}, where D_{nn} is the length of the nearest-neighbour path starting from node 1 (i.e. the path considered above), determine the pheromone levels for *all* edges e_{ij} after the first iteration, where the four ants followed the paths $(1, 4, 2, 5, 3)$, $(2, 4, 1, 3, 5)$, $(5, 3, 1, 4, 2)$, and $(1, 2, 3, 4, 5)$, respectively. (Note that, as usual, the ants also return to their start node in the final step.) For the pheromone updating rule, set the evaporation rate ρ to 0.5, and the pheromone limits $\tau_{\min}$ and $\tau_{\max}$ to 0.1 and $1/(\rho D_{\text{nn}})$, respectively. (4p)

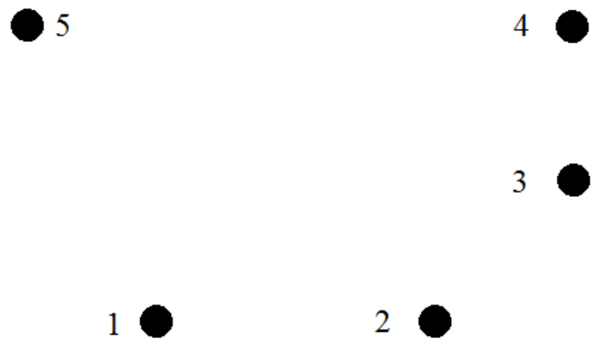

Figure 1: Construction graph for Problem 3. The nodes are located at $(1, 0)$ (node 1), $(3, 0)$ (node 2), $(4, 1)$ (node 3), $(4, 2)$ (node 4), and $(0, 2)$ (node 5).

4. The penalty method is a classical optimization method (which, however, also can be used in connection with stochastic optimization) for solving constrained minimization problems.

   (a) In the penalty method a penalty function is used for measuring the degree to which the constraints are violated for a given variable vector $\mathbf{x}$. The penalty function is then added to the objective function $f(\mathbf{x})$. Write down the general expression for the penalty function, carefully explaining all variables and parameters. (1p)

   (b) Use the penalty method to find the minimum of the function

   $$f(x_1, x_2) = (x_1 - 6)^2 + (x_2 - 7)^2, \tag{3}$$

   subject to the constraints

   $$g_1(x_1, x_2) = -3x_1 - 2x_2 + 6 \le 0, \tag{4}$$

   $$g_2(x_1, x_2) = -x_1 + x_2 - 3 \le 0, \tag{5}$$

   $$g_3(x_1, x_2) = x_1 + x_2 - 7 \le 0, \tag{6}$$

   and

   $$g_4(x_1, x_2) = \frac{2}{3}x_1 - x_2 - \frac{4}{3} \le 0, \tag{7}$$

   Hint: Start at the unconstrained minimum, and examine the constraints carefully. It is also a good idea to plot the set of feasible points before starting with the actual minimization. (4p)