Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Closed book and notes exam[1]

October 26, 2016

Time: 8.30 – 12.30

Teacher: Paolo Falcone (0761 25 7050)

Allowed material during the exam: Mathematics and physics Handbook and a Chalmers approved calculator[2,3].

The exam consists of 4 problems with a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Exam review date will be posted on the course homepage.

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[1] Textbook, personal notes and printouts of the course slides are ***not*** allowed.

[2] See https://student.portal.chalmers.se/en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx

[3] A limited number of Chalmers approved calculators are available at Madeleine Persson's office, located at the fifth floor of the E-building.

Problem 1 (10 p)

Consider the 2-step LMM

$$x_{n+2} + \alpha_1 x_{n+1} + \alpha_0 x_n = h\left(\beta_2 f_{n+2} + \beta_1 f_{n+1} + \beta_0 f_n\right)$$

(a) Write the conditions on the coefficients α_0, α_1, β_0, β_1, β_2, such that the method is convergent (2p)

(b) Calculate the coefficients α_0, α_1, β_0, β_1, β_2 such that the method is convergent with the highest order. (5p)

(c) Verify the result obtained at point (b) by simulating the system $\dot{x}(t) = -3x(t)$ and comparing the result against the exact solution. I.e., for a p-order method, you should obtain a global error $E_n = (O)(h^p)$ (3p)

Problem 2 (5 p)

(a) Explain how to identify the parameters θ_1, θ_2 for the system

$$y(t) = \theta_1 \sqrt{y(t-1)} + \theta_2 e^{u(t-1)}$$

(2p)

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(b) After collecting data to solve your system identification problem, you find out that $\hat{R}_{\varepsilon u}(\tau) \neq 0$, for a positive τ. Does this information provide any useful suggestion in the choice of the model? (3p)

Problem 3 (5 p)

Consider the system

$$\dot{x}(t) = \begin{bmatrix} -1.5 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),\ x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

with $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Calculate $x(2)$ when the system is subject to the constraint $x_1(t) - x_2(t) = u(t)$.

Verify that the constraint $x_1(t) - x_2(t) = u(t)$ is satisfied for $t = 1,\ 2$.

Problem 4 (5 p)

Mark with True or False the following statements. For the False ones provide the correct statement. The indicated points will be awarded only in case of right answer and correct statement.

1. Implicit integration methods have, in general, higher computational complexity and larger stability regions. (1p)

   True ☐ False ☐

2. In a set of identification experiments, the variance of the estimated parameters increases with the size of the data set. (1p)

   True ☐ False ☐

3. The predictor calculates the value of the output, based on previous samples of input, output and noise. (1p)

   True ☐ False ☒

4. The DAE

$$\dot{x} = f(x, y),\ 0 = g(x, y)$$

has index 1. (1p)

True ☐ False ☐

5. The prediction error method minimizes the variance of the prediction error. (1p)

True ☐ False ☐