Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Closed book and notes exam[1]

August 22, 2016

Time: 8.30 – 12.30

Teacher: Paolo Falcone (0761 25 7050)

Allowed material during the exam: Mathematics and physics Handbook and a Chalmers approved calculator[2,3].

The exam consists of 5 exercises with a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Exam review date will be posted on the course homepage.

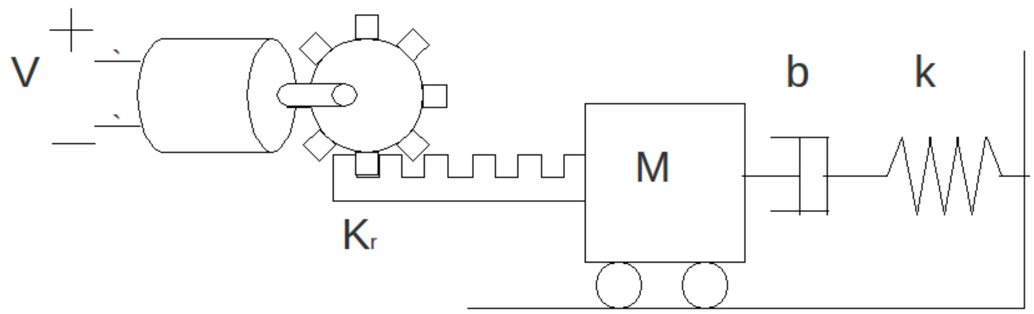
[1] Textbook, personal notes and printouts of the course slides are ***not*** allowed.

[2] See https://student.portal.chalmers.se/en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx

[3] A limited number of Chalmers approved calculators are available at Madeleine Persson's office, located at the fifth floor of the E-building.

Exercise 1 (5 p)

Consider the system in the figure below, consisting (from the left) of a DC motor, a mechanism for converting the rotating into a translating motion, a mass, a friction and elastic elements.



(a) Explain why a state space model in the form of a system of DAEs is expected for the system in the figure. (2p)

(b) Show how the state space model formulated at point **(a)** as a system of DAEs can be rewritten as a set of ODEs by smartly rearranging the system (and obtaining an equivalent one). (3p)

Exercise 2 (5 p)

(a) The system

$$y(t) = 0.6y(t-1) + 0.3u(t-1) + v(t)$$

has been used to generate N samples of input and output data. Such data is used to solve a system identification problem. The PE method is applied and the following model is chosen

$$y(t) + ay(t-1) = bu(t-1).$$

Can you guess which values do the parameters a and b converge to as $N \to \infty$? Rigorously motivate your answer. (3p)

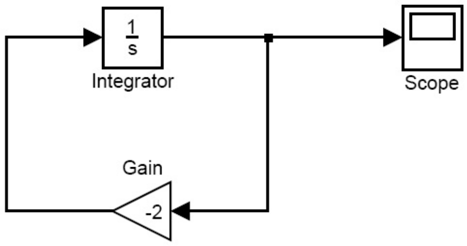
(b) The prediction error methods at point **(a)** leads to a least squares problem. Provide a necessary condition on the input signal $u(t)$ such that the least squares formula can be applied. (2p)

Exercise 3 (5 p)

Calculate the spectrum of the rotational speed of a rotating body, with a friction element of $b = 10\ kNms/rad$ and and inertia $J = 10^{-3} Kg\ m^2$, when the input torque is a white noise signal with variance 1.

Exercise 4 (5 p)

Consider the Simulink model below:



(a) You would like to simulate the system using a Runge-Kutta method

$$\begin{aligned} k_1(t) &= f(x(t)) \\ k_2(t) &= f(x(t) + \frac{h}{2}k_1(t)) \\ x(t+h) &= x(t) + hk_2(t) \end{aligned}$$

which is the largest step size that can be used in order to have a stable simulation? (4p)

(b) What are the values of $x(0.2)$ and $x(0.4)$ when using a step size of $h = 0.2$ ($x(0)$=1)? (1p)

Exercise 5 (5 p)

Mark with True or False the following statements. For the False ones provide the correct statement. The indicated points will be awarded only in case of right answer and correct statement.

1. The *correlation analysis* is a nonparametric identification method where samples of the impulse response of a system are estimated based on experimental data. (1p)

   True ☐ False ☐

2. The periodogram is a deterministic quantity. (1p)

   True ☐ False ☐

3. In the Forward Euler simulation method for ODEs, the *local error* is the error introduced by the method over the whole simulation time. (1p)

   True ☐ False ☐

4. The Prediction Error Method (PEM) is based on the application of the least squares formula. (1p)

True ☐ False ☐

5. A DAE has differentiation index m, if m is the minimal number of differentiations of the DAE that is necessary in order to solve the DAE itself. (1p)

True ☐ False ☐