MODELING AND SIMULATION EXAM

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SOLUTIONS

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Problem 1 (1)

a) The state space model is a system of DAEs because the rotating speed of the DC motor and the velocity of the mass M are constraind through a static relationship

b) The motor shaft and the mass can be lumped into an equivalent inertia thus eliminating one state variable

Problem 2

a) $a \to 0.6$ and $b \to 0.3$ as $N \to \infty$

$y(t) = \vartheta_0^T \varphi(t) + e(t)$ with $\vartheta_0 = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}$, $\varphi(t) = \begin{bmatrix} y(t-1) \\ u(t-1) \end{bmatrix}$

$y(t|\vartheta) = \vartheta^T \varphi(t)$ $\vartheta = \begin{bmatrix} a \\ b \end{bmatrix}$

$\varepsilon(t|\vartheta) = y(t) - y(t|\vartheta) = (\vartheta_0 - \vartheta)^T \varphi(t) + e(t)$

In PEM $\vartheta^* = \arg\min_{\vartheta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t|\vartheta) = \arg\min_{\vartheta} V_N(\vartheta)$

As $N \to \infty$ $V_N(\vartheta) \to \bar{E}[\varepsilon^2(t|\vartheta)]$

Hence $\vartheta^* = \arg\min_{\vartheta} \bar{E}[\varepsilon^2(t|\vartheta)]$ as $N \to \infty$

$\bar{E}[\varepsilon^2(t|\vartheta)] = \bar{E}[\varphi^T(\vartheta_0 - \vartheta)(\vartheta_0 - \vartheta)^T \varphi(t)] + \bar{E}[e^2(t)]$

The minimum is achieved with $\vartheta = \vartheta^0$. That is

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}$$

Problem 3

The system considered in this problem has TF:

$$G(s) = \frac{\omega(s)}{T(s)} = \frac{1/b}{1 + J/b\, s}$$

where $\omega(s)$ and $T(s)$ are the Laplace transforms of the rotational speed and the input torque, respectively.

The spectrum $\Phi_\omega(\omega)$ of ω is:

$$\Phi_\omega(\omega) = |G(j\omega)|^2 \Phi_T(\omega) = |G(j\omega)|^2 = \frac{1/b^2}{1 + J^2/b^2\, \omega^2}$$

Problem 4

a) The system is: $\dot{x} = -2x$

The generic iteration of the RK method is:

$$x_{k+1} = x_k - 2h x_k (1-h) = [1 - 2h(1-h)] x_k$$

stable iff $|1 - 2h(1-h)| < 1,\ h > 0 \Rightarrow \underline{0 < h < 1}$

b) $x_1 = x(0.2) = 1\,[1 - 2\cdot 0.2\,(1-0.2)] = 0.68$

$$x_2 = \underbrace{0.68}_{= x_1}\ [1 - 2\cdot 0.2 \cdot (1-0.2)] = 0.68^2$$

Problem 5

1. T
2. F
3. F
4. F
5. T