

MODELING AND SIMULATION EXAM

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SOLUTIONS

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Problem 1

(1)

a) The state space model is a system of DAEs because the rotating speed of the DC motor and the velocity of the mass M are constrained through a static relationship

b) The motor shaft and the mass can be lumped into an equivalent inertia thus eliminating one state variable

Problem 2

a) $a \rightarrow 0.6$ and $b \rightarrow 0.3$ as $N \rightarrow \infty$

$$y(t) = \vartheta_0^T \varphi(t) + e(t) \quad \text{with } \vartheta_0 = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}, \varphi(t) = \begin{bmatrix} \varphi(t-1) \\ u(t-1) \end{bmatrix}$$

$$y(t+1|\vartheta) = \vartheta^T \varphi(t)$$

$$\vartheta = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\varepsilon(t|\vartheta) = y(t) - y(t+1|\vartheta) = (\vartheta_0 - \vartheta)^T \varphi(t) + e(t)$$

$$\text{In PEM } \vartheta^* = \underset{\vartheta}{\text{argmin}} \frac{1}{N} \sum_{t=1}^N \varepsilon^2(t|\vartheta) = \underset{\vartheta}{\text{argmin}} V_N(\vartheta)$$

$$\text{As } N \rightarrow \infty \quad V_N(\vartheta) \rightarrow E[\varepsilon^2(t|\vartheta)]$$

$$\text{Hence } \vartheta^* = \underset{\vartheta}{\text{argmin}} E[\varepsilon^2(t|\vartheta)] \quad \text{as } N \rightarrow \infty$$

$$E[\varepsilon^2(t|\vartheta)] = E[\varphi^T (\vartheta_0 - \vartheta) (\vartheta_0 - \vartheta)^T \varphi(t) + e^2(t)]$$

The minimum is achieved with $\vartheta = \vartheta^*$. That is

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}$$

(2)

Problem 3

The system considered in this problem has

TF :

$$G(s) = \frac{\omega(s)}{T(s)} = \frac{1/b}{1 + \frac{J}{b}s}$$

where $\omega(s)$ and $T(s)$ are the Laplace transforms of the rotational speed and the input torque, respectively.

The spectrum $\Phi_{\omega}(\omega)$ of ω is:

$$\Phi_{\omega}(\omega) = |G(j\omega)|^2 \Phi_T(\omega) = |b(j\omega)|^{-2} = \frac{1/b^2}{1 + \frac{J^2}{b^2} \omega^2}$$

Problem 4

a) The system is: $\dot{x} = -2x$
The generic iteration of the RK method is:

$$x_{k+1} = x_k - 2h x_k (1-h) = (1 - 2h(1-h)) x_k$$

stable iff $|1 - 2h(1-h)| < 1, h > 0 \Rightarrow \underline{\underline{0 < h < 1}}$

$$b) x_1 = x(0.2) = 1 [1 - 2 \cdot 0.2 (1 - 0.2)] = 0.68$$

$$x_2 = \underbrace{0.68}_{x_1} [1 - 2 \cdot 0.2 \cdot (1 - 0.2)] = 0.68^2$$

||
x₁

Problem 5

1. T
2. F
3. F
4. F
5. F

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The SCARA manipulator shown in the figure is holding an object of mass m . The link masses are m_1, m_2, m_3 and m_4 . Find the joint expressions that can balance the gravity forces.



Figure 1: SCARA manipulator.

2. In a translational control system, an integral action is necessary to achieve zero position error in steady state. Is this statement true or false? (10%)

3. The h -norm of a system manipulates with the process and the system. Is this statement true or false? (10%)

$$T = \frac{1}{s} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -ms + m & -m \\ -m & -ms + m \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

4. The gain with respect to the input U of a transfer function $G(s)$ is the value of $G(s)$ at $s=0$. Is this statement true or false? (10%)

5. The gain with respect to the input U of a transfer function $G(s)$ is the value of $G(s)$ at $s=0$. Is this statement true or false? (10%)

6. The gain with respect to the input U of a transfer function $G(s)$ is the value of $G(s)$ at $s=0$. Is this statement true or false? (10%)

7. What is the primary purpose of the Laplace transform? (10%)

8. What is the objective of an impedance control system? (10%)