Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Closed book and notes exam[1]

January 7, 2016

Time: 14.00 – 18.00

Teacher: Paolo Falcone. Consultation over the phone only at 0761 25 7050

Allowed material during the exam: Mathematics and physics Handbook and a Chalmers approved calculator[2,3].

The exam consists of 4 exercises with a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Exam review date will be posted on the course homepage.

READ. IMPORTANT!!!!

1. Students who have attended the course in 2015 must solve problems 1-4 and 6.

2. Students who have attended the course in 2014 must solve problems 1-3 and 5-6

[1]Textbook, personal notes and printouts of the course slides are ***not*** allowed.

[2]See https://student.portal.chalmers.se/en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx

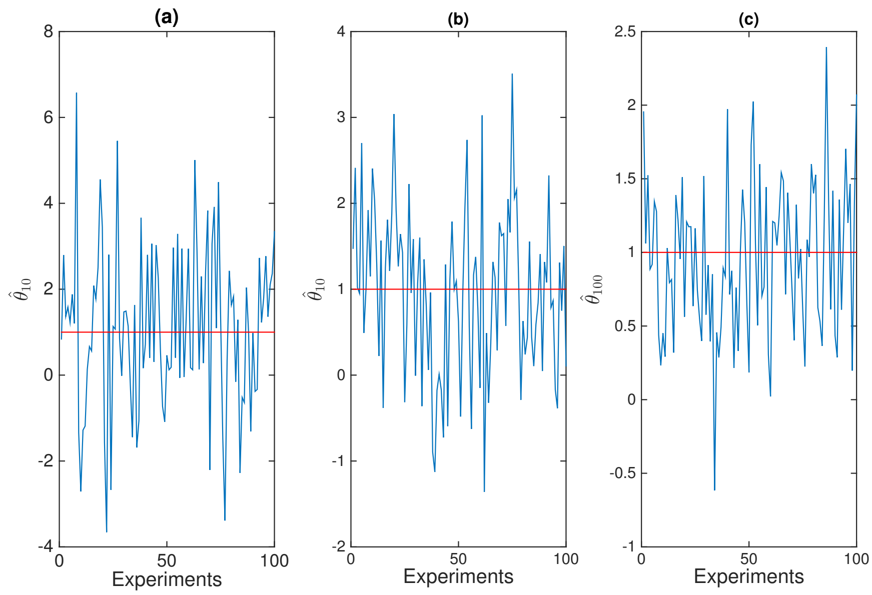
[3]A limited number of Chalmers approved calculators are available at Madeleine Persson's office, located at the fifth floor of the E-building.

Exercise 1 (5 p)

1. Give an example of physical system with four states, whose state space model results in a DAE. (3p)
2. Derive the state space model of the system at the previous point and determine the index of the DAE. (2p)

Exercise 2 (5 p)

(a) The figure below, where the red line shows the true parameter and $\hat{\theta}_N$ is the estimated parameter with a data set of size N, shows the solutions of a system identification problem solved with the PEM. In particular, at the i-th experiment, $\hat{\theta}_N$ is estimated with a new set of N input and output samples.



Explain what changes in the data sets have led tho the results in the three plots, and rigorously motivate your statements. (3p)

(b) The PE method is applied to estimate the parameters a and b of the model

$$y(t) + ay(t-1) = bu(t-1).$$

Guess which values do the parameters a and b converge to as $N \to \infty$ and by assuming that the data has been generated by the system

$$y(t) = 0.6y(t-1) + 0.3u(t-1) + v(t).$$

Rigorously motivate your answer. (2p)

Exercise 3 (5 p)

A system has to be modeled as

$$y(t) = G(q)u(t).$$

The system is excited with the input signal $u(t) \sim N(0, 0.2)$ to generate output and input signals sampled with a sampling time T.

Estimate the transfer function $G(z)$ by assuming that the real function $\dfrac{1}{2 + 2\cos\omega T}$ well approximates the spectrum of the output signal.

Exercise 4 (5 p)

Consider the IVP $\dot{x}(t) = 0,\ x(0) = 1$.

1. Simulate the system for 5 steps with the LMM

   $$x_{n+2} + 4x_{n+1} - 5x_n = h(4f_{n+1} + 2f_n).$$

   Does the result at the previous point look as expected? Motivate your answer. (2p)

2. Show that the previous LMM is not zero-stable. (3p)

Exercise 5 (5 p)

Consider the system

$$\dot{x}(t) = \begin{bmatrix} -10 & 2 \\ -2 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t),$$

with $x(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Write the generic n-th iteration calculating an approximation of the solution $x(t_n)$ for both the

1. Backward Euler method, (2p)

2. the simplest Runge-Kutta method you have studied. (3p)

Exercise 6 (5 p)

Answer the following questions and mark with True or False the following statements also providing a brief explanation. The indicated points will be awarded only in case of right answer and correct explanation.

1. The PEM holds for ARX models only. (1p)

   True ☐ False ☐

2. In the BE method, the step size must be chosen in order to ensure stability. (1p)

   True ☐ False ☐

3. The correlation analysis method estimates the samples of the impulse response. (1p)

   True ☐ False ☐

4. Which non-parametric system identification techniques would you use to have an estimate of the frequency response of a system? (1p)

5. Rigorously explain the difference between explicit and implicit numerical simulation methods. (1p)