

**ESS101 Modelling and simulation**

***Closed book and notes exam***<sup>1</sup>

August 24, 2015

**Time:** 8.30 – 12.30

**Teacher:** Paolo Falcone,

**TA:** Maliheh Sadeghi Kani, 031 772 3893

**Allowed material during the exam:** Mathematics Handbook and a Chalmers approved calculator<sup>2,3</sup>.

The exam consists of 4 exercises with a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Exam review date will be posted on the course homepage.

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<sup>1</sup>Textbook, personal notes and printouts of the course slides are *not* allowed.

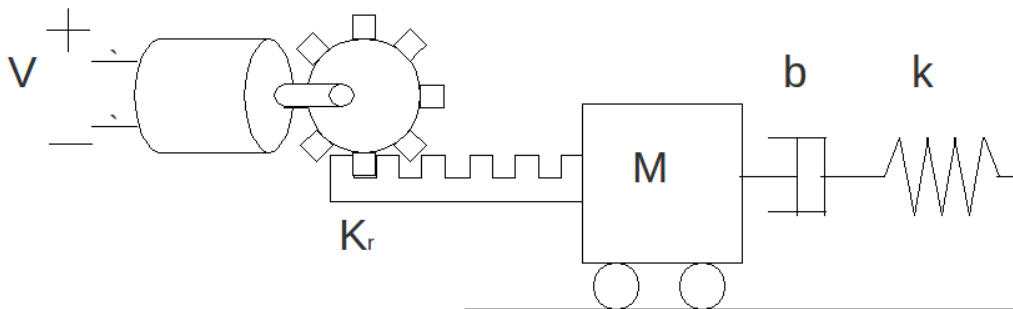
<sup>2</sup>See <https://student.portal.chalmers.se/en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx>

<sup>3</sup>A limited number of Chalmers approved calculators are available at Madeleine Persson's office, located at the fifth floor of the E-building.

**Exercise 1**

(10 p)

Consider the system in the figure below, consisting (from the left) of a DC motor, a mechanism for converting the rotating into a translating motion, a mass, a friction and elastic elements.



- (a) Explain why a causality conflict in the bond graph and a state space model in the form of a system of DAEs are expected for the system in the figure. (2p)
- (b) Draw a bond graph of the system and mark the causality. (2p)
- (c) Derive a state-space model from the obtained bond graph. (3p)
- (d) The causality conflict detected in the bond graph at point (b) can be removed and the state space model formulated at point (c) rewritten as a set of ODEs by smartly rearranging the system. Show how a conflict-free bond graph and a system of ODEs can be derived for the system in figure. (3p)

**Exercise 2**

(5 p)

Consider the system

$$y(t) + ay(t - 1) = bu(t - 1) + e(t)$$

where  $e(t)$  is white noise.

Estimate the parameters  $a$  and  $b$ , by assuming that

1. the data used for parameters identification is generated through the system

$$y(t) = 0.6u(t - 1) + 0.3u(t - 2) + v(t)$$

where  $u(t)$  and  $v(t)$  are white noises with variances 1 and 2, respectively, and uncorrelated,

2. the number of samples tends to infinite.

**Exercise 3** (5 p)

Consider the system

$$y(t) - 0.2y(t - 1) = u(t) - 0.1u(t - 1)$$

where  $u(t)$  is white noise with variance  $\lambda_u$ . Calculate

(a) the spectrum  $\phi_y(\omega)$  of  $y(t)$ . (2p)

(b) The cross spectrum  $\phi_{yu}(\omega)$ . (3p)

**Exercise 4** (5 p)

Simulate for five steps the following system with both Forward and Backward Euler methods.

$$\dot{x}(t) = \begin{bmatrix} -2 & 2 \\ 1 & 0 \end{bmatrix} x(t),$$

with  $x(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ . Motivate the choice of the integration step in both cases.