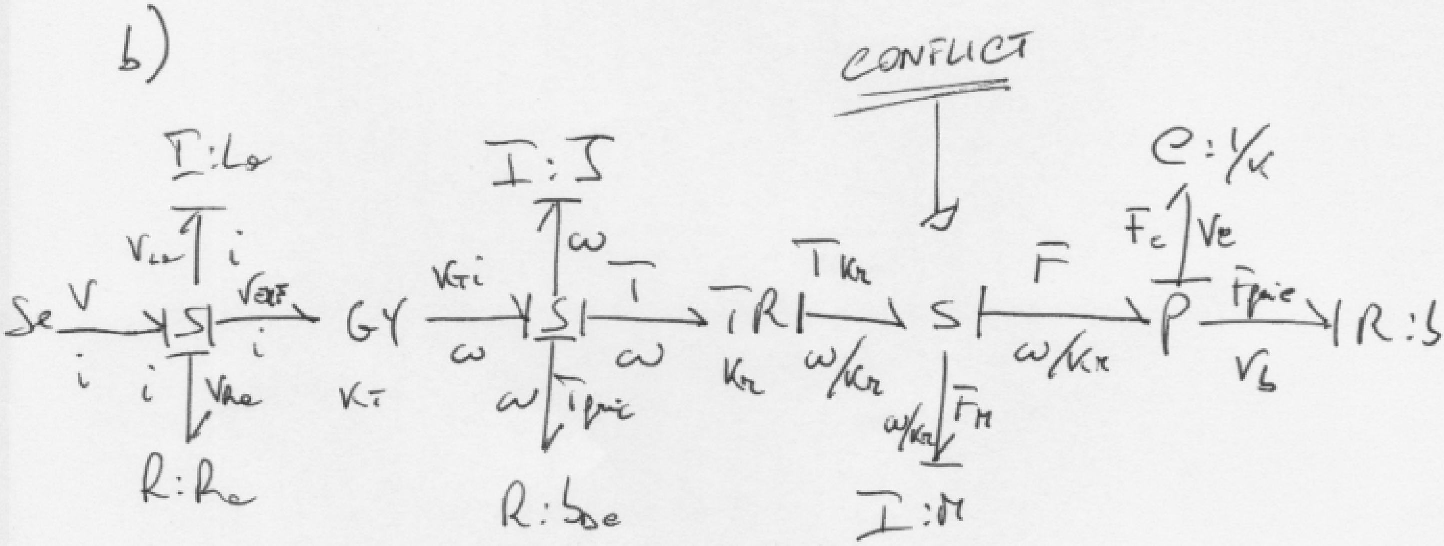
EXERCISE 1

(1)

a) The causality conflict in the bond graph asked a state space model consisting of DAE stem from the fact that the rotating speed of the DC motor and the velocity of the mass M are constrained through a static relationship.

b)



Where:

- L_a, R_a inductance, resistance of the DC-motor armature
- V_{EMF} DC motor back-emf
- J " " shaft inertia
- b_{DC} " " friction coefficient
- K_T, K_R " " Torque constants.

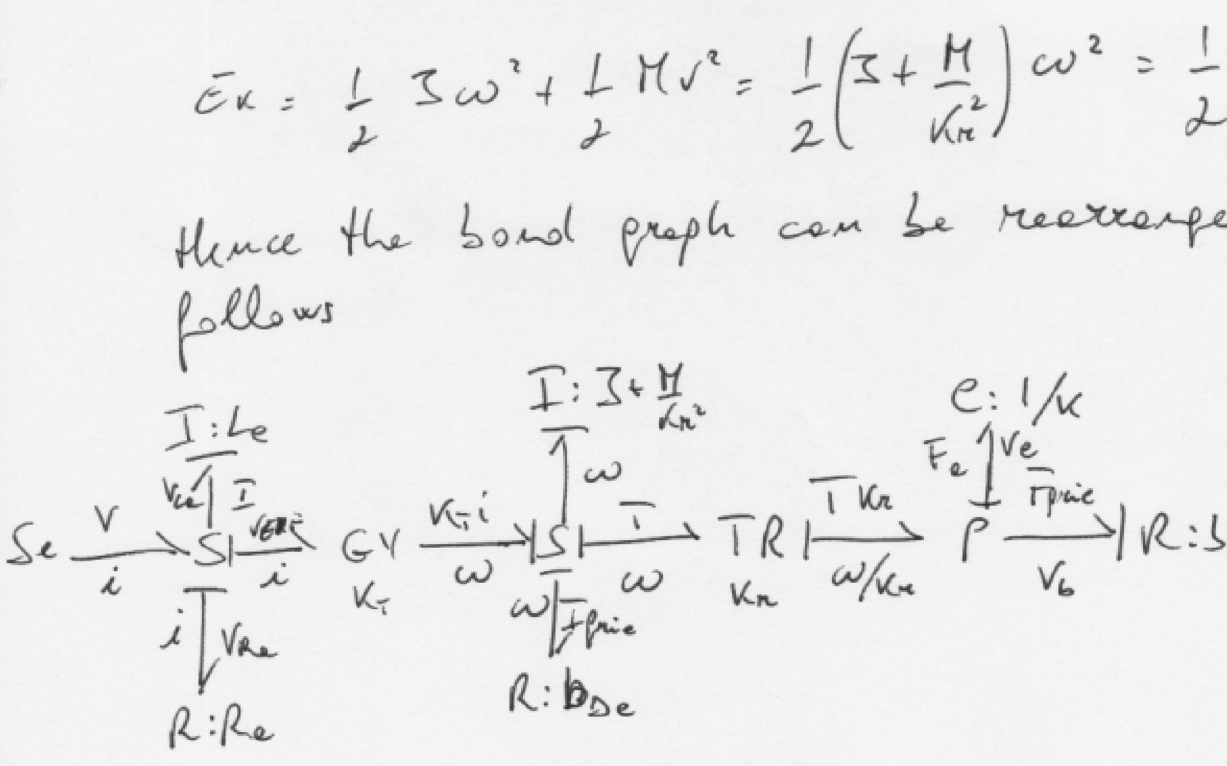
c) The resulting state space model is:

$$
\begin{cases}
i = -\dfrac{R_a}{L_a} i - \dfrac{K_v}{L_a}\omega + \dfrac{V}{L_a} \\[2mm]
\dot{\omega} = \dfrac{K_T}{J} i - \dfrac{b_{DC}\,\omega}{J} - \dfrac{T}{J} \\[2mm]
\dot{v} = \dfrac{T \cdot K_R}{M} - \dfrac{K}{M} x \\[2mm]
\dot{x} = v - \dfrac{K}{b} x \\[2mm]
\dfrac{\omega}{K_R} = v
\end{cases}
$$

d) We observe that the kinetic energy of the DC motor shaft and the mass M is:

$$E_K = \frac{1}{2} J\omega^2 + \frac{1}{2} M v^2 = \frac{1}{2}\left(J + \frac{M}{K_R^2}\right)\omega^2 = \frac{1}{2} J_{eq}\,\omega^2$$

Hence the bond graph can be rearranged as follows



The correspondig state space model is.

$$
\begin{cases}
i = -\dfrac{R_e}{L_e}\, i - \dfrac{K_v}{L_e}\, \omega + \dfrac{V}{L_e} \\[2ex]
\dot{\omega} = \dfrac{K_T}{J_{eq}}\, i - \dfrac{b_{re}}{J_{eq}}\, \omega \\[2ex]
\dot{x} = \dfrac{\omega}{K_r} - \dfrac{K}{L}\, x
\end{cases}
$$

EXERCISE 2

The prediction model is

$\hat{y}(t) = \theta^T \varphi(t)$ with $\theta^T = [a \ b]$ and $\varphi(t) = \begin{bmatrix} -y(t-1) \\ u(t-1) \end{bmatrix}$

By the LS formula:

$$
\hat{\theta}_N = \begin{pmatrix} \frac{1}{N}\sum_{t=1}^{N} y^2(t-1) & \frac{1}{N}\sum_{t=1}^{N} -y(t-1)u(t-1) \\ \frac{1}{N}\sum_{t=1}^{N} -y(t-1)u(t-1) & \frac{1}{N}\sum_{t=1}^{N} u^2(t-1) \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{N}\sum_{t=1}^{N} -y(t-1)y(t) \\ \frac{1}{N}\sum_{t=1}^{N} u(t-1)y(t) \end{pmatrix}
$$

As $N \to \infty$, $\hat{\theta}_N \to \hat{\theta}$ and

$$
\hat{\theta} = \begin{pmatrix} \bar{E}[y^2(t)] & -\bar{E}[y(t)u(t)] \\ -\bar{E}[y(t)u(t)] & \bar{E}[u(t)] \end{pmatrix}^{-1} \begin{pmatrix} -\bar{E}[y(t)y(t-1)] \\ \bar{E}[y(t)u(t-1)] \end{pmatrix} = \begin{bmatrix} -0.07 \\ 0.6 \end{bmatrix}
$$

EXERCISE 3

The discrete time TF is:

$$G(z) = \frac{1-0.1z^{-1}}{1-0.2z^{-1}} \quad \text{with } Y(z) = G(z)U(z)$$

and $Y(z)$, $U(z)$ the Z-transforms of $y(t)$ and $u(t)$, respectively.

a) $\Phi_y(\omega) = |G(i\omega)|^2 \Phi_u(\omega) = |G(i\omega)|^2 \lambda_u$

since $u(t)$ is white noise, and

$G(i\omega) = G(z)\big|_{z=e^{i\omega T}}$ with T the sampling time.

$$|G(i\omega)|^2 = G(i\omega)\overline{G(i\omega)} = \frac{1.01-0.2\cos\omega T}{1.04-0.4\cos\omega T}$$

b) $$\Phi_{yu}(\omega) = \frac{1-0.1e^{-i\omega T}}{1-0.2e^{-i\omega T}} \cdot \lambda_u$$

EXERCISE 4

The iterations for the two methods are:

$$x_{k+1} = x_k + hAx_k \quad (FE)$$

$$x_{k+1} = (I + hA)^{-1} x_k \quad (BE)$$

with $A = \begin{bmatrix} -2 & 2 \\ 1 & 0 \end{bmatrix}$ and h the integration step.

The two eigenvalues of A are $\lambda_1 = -2.7321$ and $\lambda_1 = 0.7321$. In order for the following inequality to hold

$$|1 + h\lambda_i| < 1 \quad \forall \lambda_i,$$

it must be $h \leq 0.73$