Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Closed book and notes exam[1]

October 23, 2013

Time: 8.30 – 12.30

Teacher: Paolo Falcone, 031 772 1803

Allowed material during the exam: Mathematics Handbook and a Chalmers approved calculator[2,3].

The exam consists of 4 exercises with a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Exam review date will be posted on the course homepage.

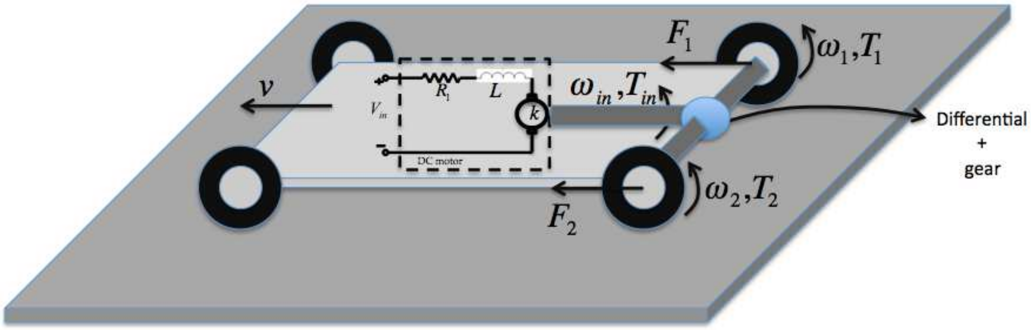
[1]Textbook, personal notes and printouts of the course slides are ***not*** allowed.

[2]See https://student.portal.chalmers.se/en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx

[3]A limited number of Chalmers approved calculators are available at Madeleine Persson's office, located at the fifth floor of the E-building.

Exercise 1 (10 p)

Consider the electrical vehicle depicted in the figure below,



with mass m and velocity v. The vehicle velocity is regulated by varying the input voltage to the electric motor V_{in}.

(a) Sketch the bond graph of the whole system by assuming that (5p)

- All shafts are infinitely stiff.
- Each rotating body is subject to viscous friction (i.e., friction torque is proportional to the angular speed).
- The differential is an "open differential". That is,

$$\omega_{in} = r\frac{\omega_1 + \omega_2}{2}, \ T_1 = T_2 = \frac{r}{2}T_{in},$$

  where r is the gear ratio of the differential.

- The vehicle is traveling uphill with a slope α.
- The vehicle is subject to air drag force proportional to v^2.

Obs. Additional physical parameters may have to be introduced depending on the proposed solution. E.g., shafts and wheels inertias, friction coefficients.

(b) Derive a state space model of the vehicle. Is the result consistent with the bond graph obtained at point **(a)**? (5p)

Exercise 2 (5 p)

(a) The system

$$y(t) = 0.6y(t-1) + 0.3u(t-1) + v(t)$$

has used to generate N samples of input and output data. Such data is used to solve a system identification problem. The PE method is applied and the following model is chosen

$$y(t) + ay(t-1) = bu(t-1).$$

Can you guess which values do the parameters a and b converge to as $N \to \infty$? Rigorously motivate your answer. (3p)

(b) The prediction error methods at point **(a)** leads to a least squares problem. Provide a necessary condition on the input signal $u(t)$ such that the least squares formula can be applied. (2p)

Exercise 3 (5 p)

A system has to be modeled as

$$y(t) = G(q)u(t).$$

The system is excited with the input signal $u(t) \sim N(0, 0.2)$ to generate output and input signals sampled with a sampling time T.

Estimate the transfer function $G(z)$ by assuming that the real function $\dfrac{1}{2 + 2\cos\omega T}$ well approximates the spectrum of the output signal.

Exercise 4 (5 p)

Mark with True or False the following statements.

1. Implicit integration methods have, in general, higher computational complexity and larger stability regions. (1p)

   True ☐ False ☐

2. In a set of identification experiments, the variance of the estimated parameters increases with the size of the data set. (1p)

   True ☐ False ☐

3. The predictor calculates the value of the output, based on previous samples of input, output and noise. (1p)

   True ☐ False ☐

4. The DAE

$$\dot{x} = f(x, y),\ 0 = g(x, y)$$

   has index 1. (1p)

   True ☐ False ☐

5. The prediction error method minimizes the variance of the prediction error. (1p)

True ☐ False ☐