SOLUTION OF THE MODELING AND SIMULATION EXAM (ESS 101)

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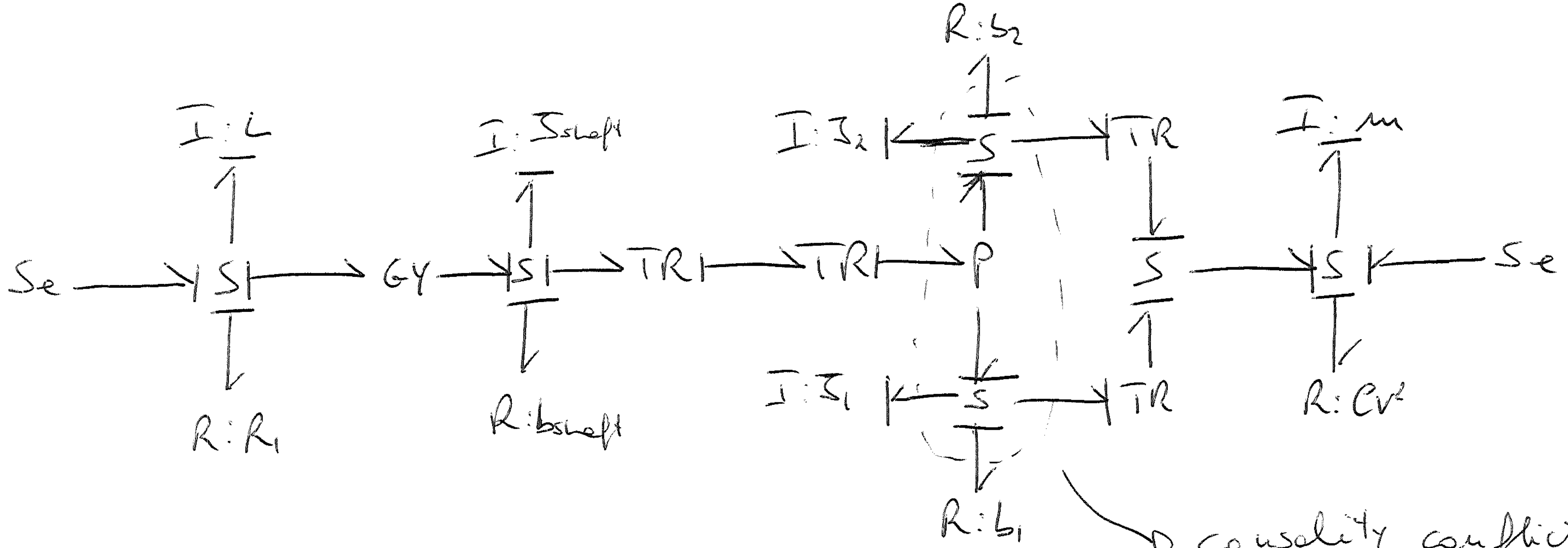
Ex 1.a)

(1)



→ causality conflicts.

$\omega_{in}, \omega_1, \omega_2, v$ can be replaced by one state variable.

Ex 1.b)

$$\begin{cases} \dot{i} = -\dfrac{R_1}{L} i - K\omega \\[2mm] \dot{\omega}_{in} = -\dfrac{b_{in}}{J_{in}} + Ki - T_{in} \\[2mm] \dot{\omega}_1 = \dfrac{T_{in} r}{2 J_1} - \dfrac{b_1}{J_1}\omega_1 - F_1 r_1 \\[2mm] \dot{\omega}_2 = \dfrac{T_{in} r}{2 J_2} - \dfrac{b_2}{J_2}\omega_2 - F_2 r_2 \\[2mm] \dot{v} = \dfrac{F_1 + F_2}{m} - \dfrac{\rho v^2}{m} - g \sin\alpha \\[2mm] v = \omega_1 r_1 = \omega_2 r_2 \\[2mm] \omega_{in} = \dfrac{r}{2}(\omega_1 + \omega_2) \\[2mm] F_1 = F_2 = \dfrac{r}{2} T_{in} \end{cases}$$

Ex 2.9)

$a \to -0.6$ and $b \to 0.3$ as $N \to \infty$

$$y(t) = \vartheta_0^T \varphi(t) + e(t) \qquad \text{with } \vartheta_0 = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix} \quad \varphi(t) = \begin{bmatrix} y(t-1) \\ u(t-1) \end{bmatrix}$$

$$\hat{y}(t|\vartheta) = \vartheta^T \varphi(t) \qquad \vartheta = \begin{bmatrix} -a \\ b \end{bmatrix}$$

$$\varepsilon(t|\vartheta) = y(t) - \hat{y}(t|\vartheta) =$$

$$= (\vartheta_0 - \vartheta)^T \varphi(t) + e(t)$$

In PEM $\vartheta^* = \underset{\vartheta}{\arg\min} \ \frac{1}{N} \sum_{t=1}^{N} \varepsilon^2(t|\vartheta)$

$$= \underset{\vartheta}{\arg\min} \ V_N(\vartheta)$$

As $N \to \infty$ $\quad V_N(\vartheta) \to \bar{E}[\varepsilon^2(t|\vartheta)]$

Hence as $N \to \infty$ $\quad \vartheta^* = \underset{\vartheta}{\arg\min} \ \bar{E}[\varepsilon^2(t|\vartheta)]$

$$\bar{E}[\varepsilon^2(t|\vartheta)] = \bar{E}[\varphi^T (\vartheta_0 - \vartheta)(\vartheta_0 - \vartheta)^T \varphi(t) +$$

$$+ 2(\vartheta_0 - \vartheta)^T \varphi(t) e(t) + e(t)] =$$

$$= \bar{E}[\varphi^T (\vartheta_0 - \vartheta)(\vartheta_0 - \vartheta)^T \varphi(t)] + \bar{E}[e^2(t)]$$

The smallest $\bar{E}[\varepsilon^2(t|\vartheta)]$ is $\bar{E}[e^2(t)]$ and is achieved with $\vartheta^* = \vartheta^0$. That is $\begin{bmatrix} -a \\ b \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.3 \end{bmatrix}$

Ex 2.b)

The least squares formula is:

$$\theta^* = R_N^{-1} f_N \quad \text{with} \quad R_N = \frac{1}{N}\sum_{t=1}^{N} \begin{bmatrix} y^2(t-1) & u(t-1)y(t-1) \\ y(t-1)u(t-1) & u^2(t-1) \end{bmatrix}$$

$$f_N = \frac{1}{N}\sum_{t=1}^{N} \begin{bmatrix} y(t-1)\,y(t) \\ u(t-1)\,y(t) \end{bmatrix}$$

$$R_N \text{ invertible} \Rightarrow E[u^2(t-1)] = R_u(0) \neq 0$$

Ex 3

Set $H(\omega) = \dfrac{1}{2+2\cos\omega t}$. Since $H(\omega) \simeq \Phi_y(\omega)$

it holds that

$$H(\omega) \simeq |G(j\omega)|^2\, \Phi_u(\omega) \quad \text{with } \Phi_u(\omega) = 0.2T$$

Hence:

$$|G(j\omega)|^2 \simeq \frac{1}{(2+2\cos\omega T)\,0.2T}$$

$G(j\omega)$ is of the type $G(j\omega) = \dfrac{1+\sum_{k=0}^{M_{num}} a_k e^{-kj\omega T}}{\left(1+\sum_{k=0}^{M_{den}} b_k e^{-kj\omega T}\right)\sqrt{0.2T}}$

Setting $M_{num}=0$, $a_0=0$, $M_{den}=1$, $b_1=1$ solves the problem.

Ex 4

1) T
2) F
3) F
4) T
5) F