Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Closed book and notes exam[1]

August 26, 2013

Time: 8.30 – 12.30

Teacher: Paolo Falcone,

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Allowed material during the exam: Mathematics Handbook and a Chalmers approved calculator[2,3].

The exam consists of 4 exercises with a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Exam review date will be posted on the course homepage.

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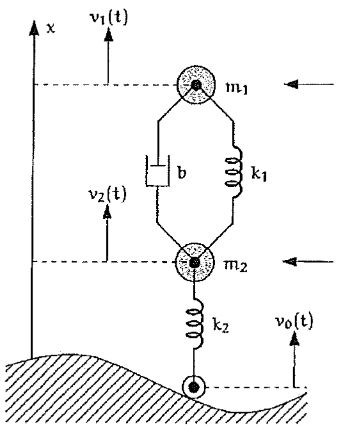
[1] Textbook, personal notes and printouts of the course slides are ***not*** allowed.

[2] See https://student.portal.chalmers.se/en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx

[3] A limited number of Chalmers approved calculators are available at Madeleine Persson's office, located at the fifth floor of the E-building.

Exercise 1 (10 p)

Consider the car suspension system in the figure below,



where m_1 and m_2 are the masses of the vehicle and the wheel, respectively, k_1 and $b = b(u)$ are the spring and friction constants, respectively, of the suspension, u is an exogenous signal and k_2 is the spring constant of the wheel.

(a) Derive a state space model of the whole suspension system. (5p)

(b) Derive a state space model of the suspension system under the simplification of rigid wheel. (2p)

(c) For the model at point **(b)** of this exercise, assume $\dot{v}_0(t)$ is a white noise with variance 1. Compute the spectrum of the signal $\dot{v}_1(t)$. (3p)

Exercise 2 (5 p)

(a) Consider the system

$$y(t) = 0.5u(t-1) + \frac{1}{1 + dz^{-1}}\xi(t)$$

where $\{\xi(t)\}$ is white noise with variance λ^2. Show how the least squares method can be applied to estimate the parameter d. (3p)

(b) Consider the system

$$y(t) = ay(t-1)^2 + b_1 u(t-1) + b_2 u(t-2)^3 + \xi(t)$$

where $\{\xi(t)\}$ is white noise with variance λ^2. Show how the least squares method can be applied to estimate the parameters a, b_1 and b_2. (2p)

Exercise 3 (5 p)

(a) What is the (differentiation) index of the following DAE? (2p)

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2 x_1 \\ \dot{x}_2 &= -x_1^2 + x_2 x_3 \\ 0 &= x_1^2 + x_2 + x_3^2. \end{aligned}$$

Motivate the answer.

(b) Show an example of nonlinear DAE with index 2. (2p)

(c) What is the index of the following linear DAE? (1p)

$$\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 12 & 6 & 0 & 0 \\ 18 & 18 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Exercise 4 (5 p)

Answer to the following questions.

1. How can the spectrum of a signal be estimated by using N samples? (1p)

2. How do the local and the global errors of the Forward Euler method depend on the step size h? (1p)

3. Explain how, in the PE method, the covariance of the estimated vector of parameters depends on the variance of the measurement noise and the size of the data set. (1p)

4. Consider the DAE

$$E\dot{x} + Ax + Bu = 0,$$

with E singular. Explain how to use the result stated in the Kronecker theorem in order to easily calculate its differentiation index. (2p)