Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Closed book and notes exam[1]

October 22, 2012

Time: 14.00 – 18.00

Teacher: Paolo Falcone,

TA: Azita Dabiri, 772 1820

Allowed material during the exam: Mathematics Handbook and a Chalmers approved calculator[2,3].

The exam consists of 4 exercises with a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Exam review date will be posted on the course homepage and scheduled no later than **November 16**th.

[1] Textbook, personal notes and printouts of the course slides are ***not*** allowed.

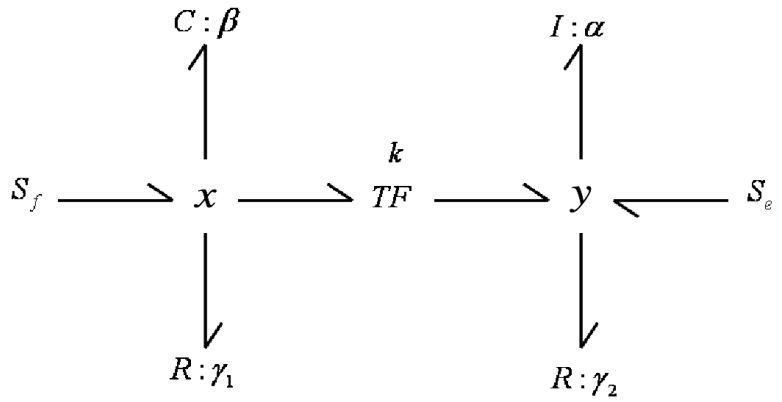
[2] See https://student.portal.chalmers.se/en/chalmersstudies/Examinations/Pages/Examinationroominstructions.aspx

[3] A limited number of Chalmers approved calculators are available at Madeleine Persson's office, located at the fifth floor of the E-building.

Exercise 1 (5 p)

(a) Sketch a second order (i.e., with two states) electrical system with a causality conflict. Concisely motivate your and check the presence of the causality conflict by drawing the bond graph of the system you have proposed. (2p)

(b) Consider the bond graph in the figure below.



List all possible combinations of junctions 'x' and 'y' leading to a conflict-free graph and mark the causality. (1p)

(c) Derive a state space model from the graph obtained at point (b). (2p)

Exercise 2 (5 p)

Consider the system

$$y(t) = 0.1u(t-1) + \frac{\xi(t)}{1 + bq^{-1} + cq^{-2}}, \;\; \xi(t) \in WN(0, \lambda).$$

Show how the parameters b and c can be estimated by using the LS method.

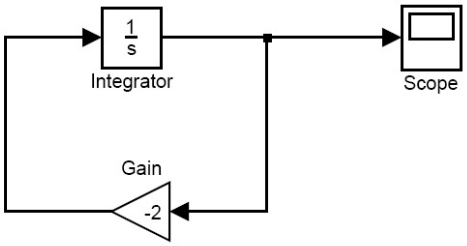
Exercise 3 (5 p)

(a) Show that the $\Phi_w(\omega) = \lambda T$, where $w(t) \in WN(0, \lambda)$ is a signal sampled with a sampling time T. (3p)

(b) Calculate the spectrum of the rotational speed of a rotating body, with a friction element of $b = 10\ kNms/rad$ and inertia $J = 10^{-3} Kg\ m^2$, when the input torque is $w(t)$ at point (a) with $\lambda = 1$. (2p)

Exercise 4 (5 p)

Consider the Simulink model below:



(a) You would like to simulate the system using a Runge-Kutta method

$$k_1(t) = f(x(t))$$
$$k_2(t) = f(x(t) + \frac{h}{2}k_1(t))$$
$$x(t+h) = x(t) + hk_2(t)$$

which is the largest step size that can be used in order to have a stable simulation? (4p)

(b) What value on $x(0.2)$ and $x(0.4)$ do you get when using a step size of $h = 0.2$ ($x(0)$=1)? (1p)

Exercise 5 (5 p)

Answer to the following questions.

1. How can the spectrum of a signal be estimated by using N samples? (1p)

2. How do the local and the global errors of the Forward Euler method depend on the step size h? (1p)

3. Explain how, in the PE method, the covariance of the estimated vector of parameters depends on the variance of the measurement noise and the size of the data set. (1p)

4. Consider the DAE

$$E\dot{x} + Ax + Bu = 0,$$

with E singular. Explain how to use the result stated in the Kronecker theorem in order to easily calculate its differentiation index. (2p)