SOLUTIONS OF THE

ESS101 - MODELING AND SIMULATION EXAM

Examination date : 22/10/2012

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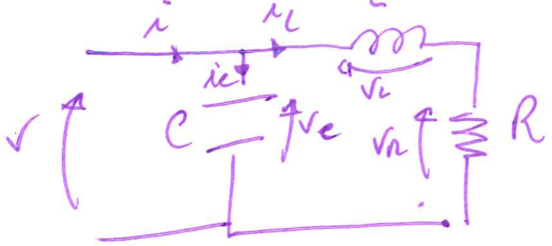
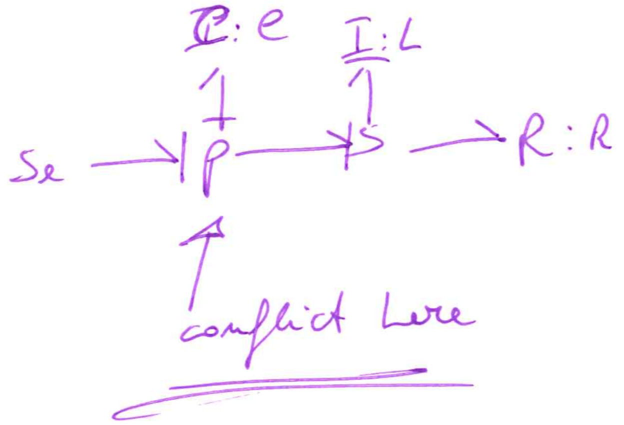
(1)

1.a)



The corresponding bond graph has a causality conflict because the voltage v_C (a state variable) is constrained to be equal to the input voltage V.

The bond graph is:



1.b) The possible combinations leading to causality conflicts-free graphs are $x = s, y = p$ and $x = p, y = s$

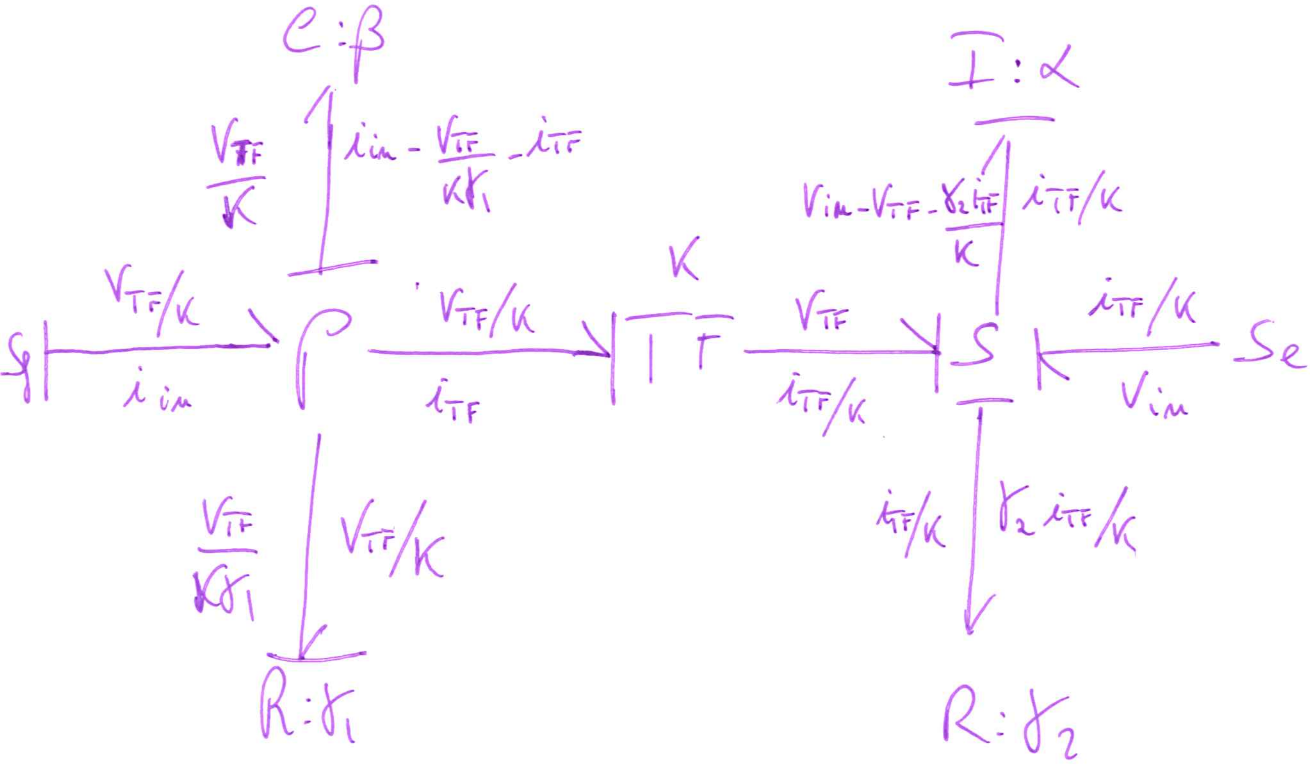
1.c) Deriving the state space model for the combination $x = s, y = p$ is trivial since the flow and effort sources are in series and parallel to the C and I elements, respectively

(2)

Hence, by setting
$$\begin{cases} x_1 & \text{effort at the C element} \\ x_2 & \text{flow at the I element} \\ f_{in} & \text{input flow} \\ e_{in} & \text{input effort} \end{cases}$$

we have
$$\begin{cases} \dot{x}_1 = \frac{1}{\beta} \cdot f_{in} \\ \dot{x}_2 = \frac{1}{\alpha} e_{in} \end{cases}$$

For the combination $x = P$, $y = S$,



By setting
$$\begin{cases} x_1 = V_{TF} \\ x_2 = i_{TF} \\ u_1 = i_{in} \\ u_2 = V_{in} \end{cases}$$
the resulting state space model is:
$$\begin{cases} \dot{x}_1 = -\frac{x_1}{\gamma_1 \cdot \beta} - x_2 \cdot \frac{K}{\beta} + u_1 \cdot \frac{K}{\beta} \\ \dot{x}_2 = -x_1 \cdot \frac{K}{\alpha} - x_2 \cdot \frac{\gamma_2}{\alpha} + u_2 \cdot \frac{K}{\alpha} \end{cases}$$

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2) The system can be rewritten as:

$$y(t) = b\,[0.1u(t-2) - y(t-1)] + e\,[0.1u(t-3) - y(t-3)] +$$
$$+ 0.1u(t-1) + \xi(t)$$

The corresponding predictor is:

$$\hat{y}(t|\vartheta) = b\,[0.1u(t-2) - y(t-1)] + e\,[0.1u(t-3) - y(t-3)] +$$
$$+ 0.1u(t-1)$$

with $\vartheta = \begin{bmatrix} b \\ e \end{bmatrix}$

Define $z(t) = y(t) - 0.1u(t-1)$. Hence,

$$\hat{z}(t|\vartheta) = \vartheta^T \varphi(t) \quad \text{with} \quad \varphi(t) = \begin{bmatrix} 0.1u(t-2) - y(t-1) \\ 0.1u(t-3) - y(t-3) \end{bmatrix}$$

The LS formula can then be used to calculate ϑ.

3.a) The spectrum for a stochastic signal $w(t)$ is defined as:

$$\Phi_w^T(\omega) = T \sum_{k=-\infty}^{k=\infty} R_w(kT) e^{-i\omega kT}$$

Since $w(t) \in WN(0,\lambda) \Rightarrow R_w(\tau) = 0 \;\; \forall \tau \neq 0$. Hence,

$$\underline{\Phi_w^T(\omega) = T \cdot \lambda}$$

(2)

3.b) The system has transfer function

$$G(s) = \frac{1/b}{1 + \frac{J}{b} \cdot s} = \frac{\Omega(s)}{T(s)}$$

$\Omega(s)$ → $\mathcal{L}$-trasform of the rotational speed

$T(s)$ → $\mathcal{L}$-trasform of the input torque.

The spectrum $\Phi_\omega(\omega)$ is

$$\Phi_\omega(\omega) = |G(j\omega)|^2 \Phi_T(\omega) = |G(j\omega)|^2 = \frac{1/b^2}{1 + \frac{J^2}{b^2}\omega^2}$$

4.a) The system that has to be simulated is:

$$\dot{x} = -2x$$

The generic iteration of the RK method can be rewritten as:

$$x_{k+1} = x_k - 2h\, x_k (1-h) = [1 - 2h(1-h)]\, x_k$$

stable iff $|1 - 2h(1-h)| < 1$, $h > 0$

The above inequalities result in $\underline{0 < h < 1}$

4.b) $x_1 = x(0.2) = 1 \cdot [1 - 2 \cdot 0.2 \cdot (1 - 0.2)] = 0.68$

$$x_2 = \underbrace{0.68}_{= x(1)} [1 - 2 \cdot 0.2 \cdot (1 - 0.2)] = 0.68^2$$

(5)

5.1) The estimate of the spectrum based on N samples is

$$\hat{\Phi}^T_{w,N} = T\,|W_N^T(\omega)|^2 \quad \text{with} \quad W_N^T(\omega) = \sum_{k=1}^{N} w(k)\, e^{-i\omega kT}$$

5.2) For the FE method

$$e_n \sim O(h^2) \quad \text{(local error)}$$
$$E_n \sim O(h) \quad \text{(global error)}$$

wher h is the integration step.

5.3) In the PE method the covariance of the estimate increases with the variance of the measurements noise and decreases as the size N of the data set increases.

5.4) Given the DAE

$$E\dot{x} + Ax + Bu = 0,$$

First, we have to check whether (E, A) is a regular matrix pencil. I.e., if $\exists \lambda : |\lambda E - A| \neq 0$.
If this is the case, the Kronecker theorem guarantees the two regular matrices U and V can be found such that:

$$UEV = \begin{pmatrix} I & 0 \\ 0 & N \end{pmatrix} \quad UAV = \begin{pmatrix} C & 0 \\ 0 & I \end{pmatrix}$$

with N Jordan block matrix to the eigenvalue 0

(6) Then, we have to calculate the nilpotency index of N. I.e., the smallest μ : $N^{\mu-1} \neq 0$ and $N^{\mu} = 0$

A well known results states that, for a linear DAE, forming a regular matrix pencil, the differentiation index is equal to the nilpotency index.