Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Examination date January 10, 2011

Time: 14.00 – 18.00

Teacher: Paolo Falcone, 772 1803

TA: Roozbeh Kianfar, 772 5793

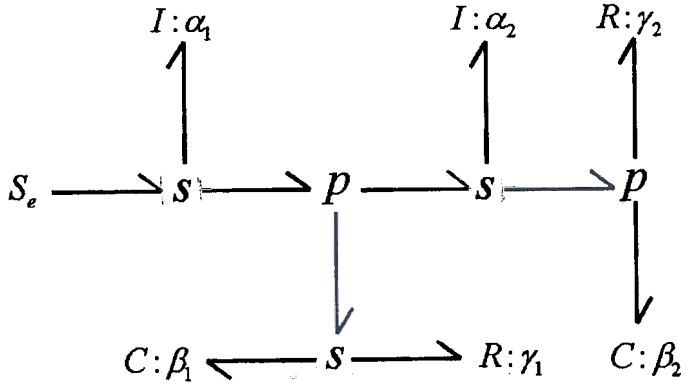
Allowed material during the exam: Mathematics Handbook and small calculator (not a PC).

The exam consists of 4 exercises with a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

You can discuss with teacher and TAs the grading of your exam on January 24th at 14.30-15.30 at the Department of Signals and Systems.

Exercise 1

Consider the following bond graph,



(a) Mark the causality.

(b) Derive a state space model.

(c) Sketch an electrical system corresponding to the bond graph above.

(d) Sketch a mechanical system corresponding to the bond graph above. (3p)

Exercise 2

Assume the system

$$y(t) = -0.5y(t-1) + u(t) + e(t),$$

where $e(t) \sim N(0,1)$, is generating the input-output data in a system identification experiment.
Choose a model parametrization and, by assuming that $N \longrightarrow \infty$, i.e., the number of data samples tends to infinity and $u(t) \sim N(0,2)$, estimate the parameters of the chosen parametrization through the least squares method. Briefly comment the choice of the parametrization and the results you obtained.

Hint. Recall that the least squares formula is

$$\theta_N^* = \left(\sum_{t=1}^{N} \varphi(t)\varphi^T(t) \right)^{-1} \left(\sum_{t=1}^{N} \varphi(t)y(t) \right)$$

Exercise 3

We want to simulate the system

$$\dot{x}(t) = \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),$$

with a step size $h = 1$.

(a) What is the simplest simulation method you would use, guaranteeing numerical stability? (2p)

(b) Write a generic iteration (i.e., $x_k = f(x_{k-1})$) of the Forward and Backward Euler methods for the above system. (3p)

Exercise 4

Mark with True or False the following statements and provide a brief explanation for the False ones.

1. Implicit integration methods have, in general, higher computational complexity and larger stability regions. (1p)

   True ☐ False ☐

2. In a set of identification experiments, the variance of the estimated parameters increases with the size of the data set. (1p)

True ☐ False ☐

3. The predictor calculates the value of the output, based on previous samples of input, output and noise. (1p)

True ☐ False ☐

4. The DAE

$$\dot{x} = f(x, y),\ 0 = g(x, y)$$

has index 1. (1p)

True ☐ False ☐

5. The prediction error method minimizes the variance of the prediction error. (1p)

True ☐ False ☐