SOLUTIONS OF THE

ESS101 - MODELING AND SIMULATON

EXAM

Examination date: 17/08/2010

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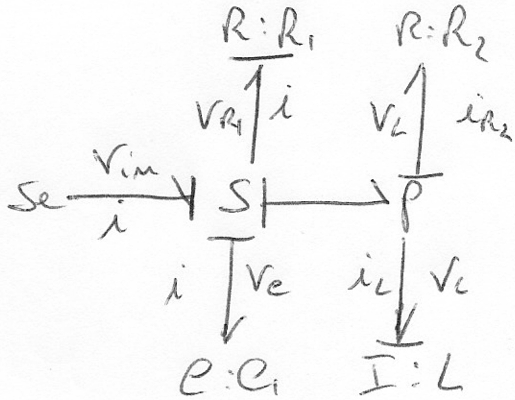
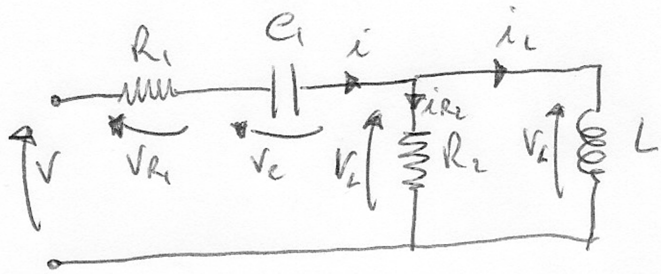
Exercise 1

a)



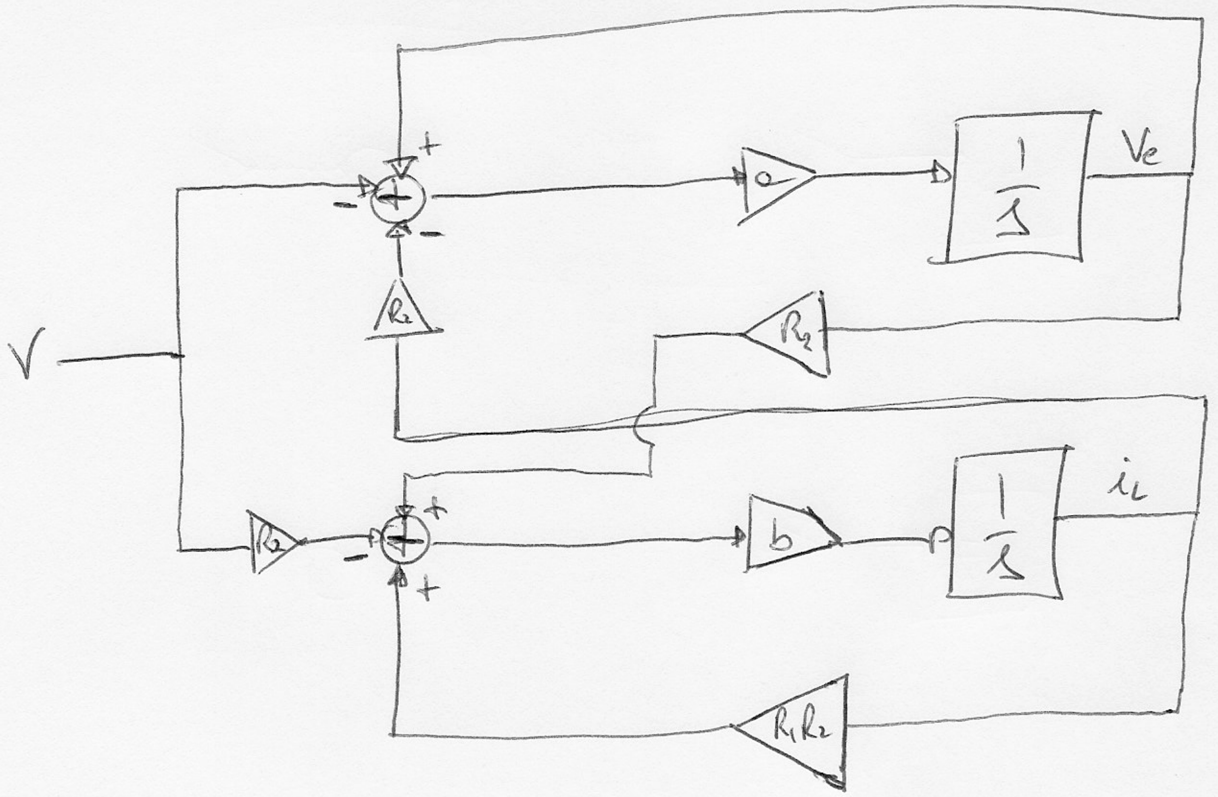
b) In order to derive a blocks diagram of the considered system, we first formulate a state space model



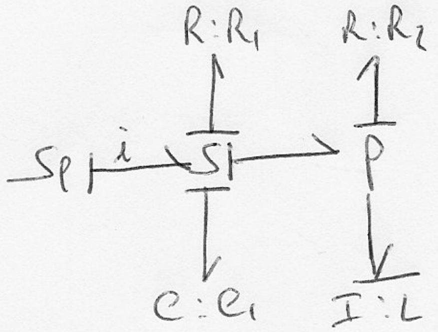
$$\begin{cases} V = R_1 i + V_e + V_L \\ i = i_{R_2} + i_L \\ i = C_1 \dfrac{dV_e}{dt} \\ V_L = L \dfrac{di_L}{dt} \end{cases} \Rightarrow \begin{cases} \dfrac{dV_e}{dt} = -\dfrac{1}{C_1(R_1+R_2)}\left[V_e - R_2 i_L - V\right] \\ \\ \dfrac{di_L}{dt} = -\dfrac{1}{L(R_1+R_2)}\left[R_2 V_e + R_1 R_2 i_L - R_2 V\right] \end{cases}$$

We define the following ~~xxxx~~ parameters:

$$a = -\frac{1}{C_1(R_1+R_2)} \quad , \quad b = -\frac{1}{L(R_1+R_2)}$$

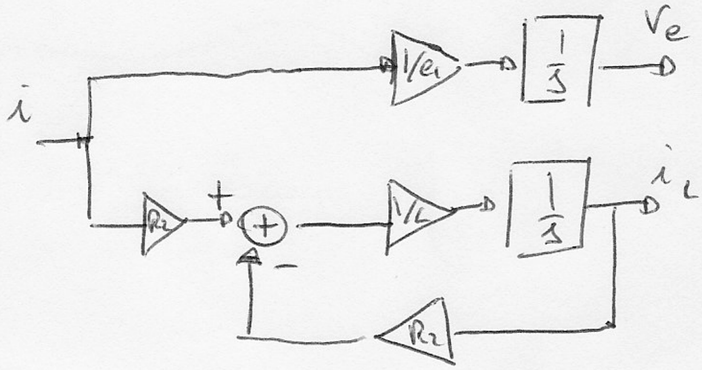


c) By replacing the voltage source with a current source the bond graph is:



and the blocks diagram becomes

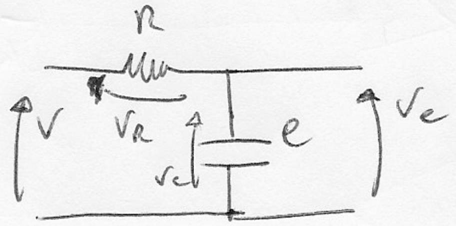
$$\begin{cases} \dfrac{dV_e}{dt} = \dfrac{i}{C_1} \\[2ex] \dfrac{di_L}{dt} = -\dfrac{R_2}{L} i_L + \dfrac{R_2}{L} i \end{cases}$$



d) The bond graph method is more convenient because of its object oriented nature. This does not require to re-write the model equation.

Exercise 2

Consider the RC-series below:



The state space model is:

$$\dot{V}_C = -\frac{1}{RC} V_C + \frac{1}{RC} V$$

and the corresponding transfer function is:

$$G(s) = \frac{V_C(s)}{V(s)} = \frac{1}{1+sRC}$$

where $V_C(s)$ and $V(s)$ are the Laplace transforms of $V_C(t)$ and $V(t)$, respectively.

The spectrum $\Phi_{V_C}(\omega)$ of the voltage across the capacitor is:

$$\Phi_{V_C}(\omega) = |G(j\omega)|^2 \, \Phi_V(\omega) = |G(j\omega)|^2 =$$

$$= \cancel{\frac{RC\,j\omega}{1+R^2C^2\omega^2}} \quad \frac{1}{1+R^2C^2\omega^2}$$

Exercise 3

a) The state space model underlying the considered blocks diagram is:

$$\begin{cases} \dot{x}_1 = -a x_1^2 + b x_2 + u \\ \dot{x}_2 = -\dfrac{x_1}{x_2} + b x_2 + u^2 \end{cases}$$

b) The modelica code implementing the considered model is

```
function model
    input Real u, "input";
    output Real y1, y2, "output";
    parameter Real a = 1;
    parameter Real b = 1;
    Real x1, x2
equation
    der(x1) = -a * x1^2 + b * x2 + u;
    der(x2) = -x1/x2 + b * x2 + u;
    y1 = x1;
    y2 = x2;
end model
```

Exercise 4

The prediction model is given by

$\hat{y}(t) = \theta^T \varphi(t)$ with $\theta = [a, b]$ and

$$\varphi(t) = \begin{bmatrix} -y(t-1) \\ u(t-1) \end{bmatrix}$$

By using the least squares method, the estimate of the parameters i given by:

$$\hat{\theta}_N = \left(\frac{1}{N} \sum_{t=1}^{N} \varphi(t) \varphi^T(t) \right)^{-1} \cdot \frac{1}{N} \sum_{t=1}^{N} \varphi(t) y(t) =$$

$$= \begin{pmatrix} \frac{1}{N} \sum_{t=1}^{N} y^2(t-1) & \frac{1}{N} \sum_{t=1}^{N} -y(t-1) u(t-1) \\ \frac{1}{N} \sum_{t=1}^{N} -y(t-1) u(t-1) & \frac{1}{N} \sum_{t=1}^{N} u^2(t-1) \end{pmatrix}^{-1} \cdot$$

$$\cdot \begin{pmatrix} \frac{1}{N} \sum_{t=1}^{N} -y(t-1)\, y(t) \\ \frac{1}{N} \sum_{t=1}^{N} u(t-1)\, y(t) \end{pmatrix}$$

If $u(t)$ and $y(t)$ are stationary, as $N \to \infty$ $\hat{\theta}_N \to \hat{\theta}$

$$\hat{\theta} = \begin{pmatrix} E[y^2(t)] & -E[y(t) u(t)] \\ -E[y(t) u(t)] & E[u(t)] \end{pmatrix}^{-1} \begin{pmatrix} -E[y(t) y(t-1)] \\ E[y(t)\, u(t-1)] \end{pmatrix} = \begin{pmatrix} -0.07 \\ 0.6 \end{pmatrix}$$