

**ESS101 Modelling and simulation**  
Examination date 100111

***Time:*** 14.00 – 18.00

***Teacher:*** Paolo Falcone, 772 1803

***Allowed material during the exam:*** Mathematics Handbook and small calculator (not a PC).

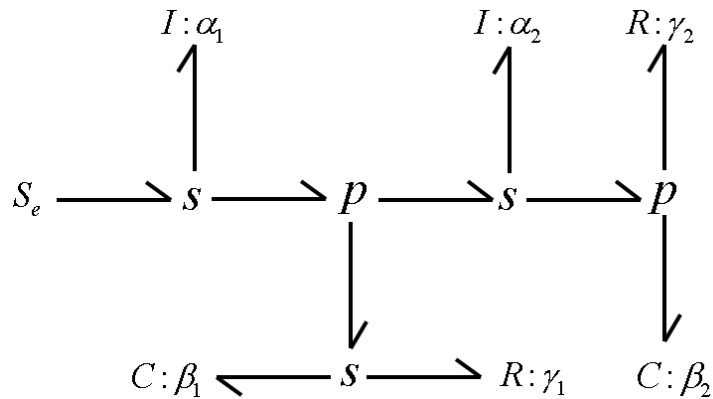
The exam consists of 4 exercises of a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Results are announced on the notice board at the latest Jan 19. You can discuss with teacher and TAs the grading of your exam on Jan 20 at 14.30-15.30 at the Department of Signals and Systems.

**Exercise 1**

(10 p)

Consider the bond graph in the figure below.



- (a) Mark the causality. (1p)
- (b) Derive a state space model. (3p)
- (c) Sketch an electrical system that can be modeled with the bond graph in figure. (3p)
- (d) Sketch an mechanical system that can be modeled with the bond graph in figure. (3p)

**Exercise 2**

(5 p)

Consider the system

$$y(t) - 0.2y(t - 1) = u(t) - 0.1u(t - 1)$$

where  $u(t)$  is white noise with variance  $\lambda_u$ . Calculate

- (a) the spectrum  $\phi_y(\omega)$  of  $y(t)$ . (2p)
- (b) The cross spectrum  $\phi_{yu}(\omega)$ . (3p)

**Exercise 3**

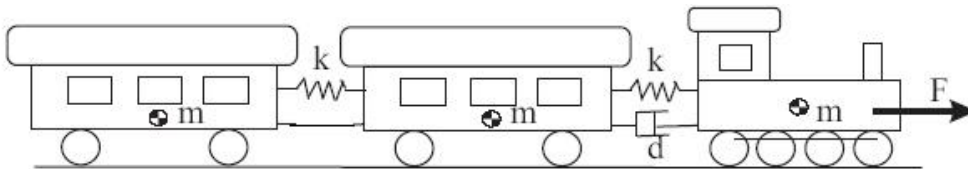
(5 p)

- (a) What type of system is implemented by the the modelica code below? Write the input output equations of the system. (2p)

```

function system
  input Real y_prev[3], u_prev[2], "input";
  output Real y, "output";
  parameter Real a[]={1,2,3}, "A";
  parameter Real b[]={1,2}, "B";
algorithm
  for i in 1:size(a,1) loop
    dummy1:=dummy1+a[i]*y_prev[i];
  end for;
  for i in 1:size(b,1) loop
    dummy1:=dummy1+b[i]*u_prev[i];
  end for;
  y:= dummy1 + dummy2;
end filter;

```



(b) Write the modelica code implementing the model of the mechanical system sketched in the above figure. (3p)

**Exercise 4** (5 p)

(a) As well known, the parameters estimate in a system identification problem, solved through the prediction error method, is a stochastic process. Describe the relationship between the variance error of the parameters estimate and (i) the number of measured data samples  $N$  and (ii) the variance  $\lambda$  of the noise in the data. (3p)

(b) What is the bias error in the parameter estimate? (2p)