SOLUTIONS OF THE MODELING AND SIMULATION EXAM (ESS101)

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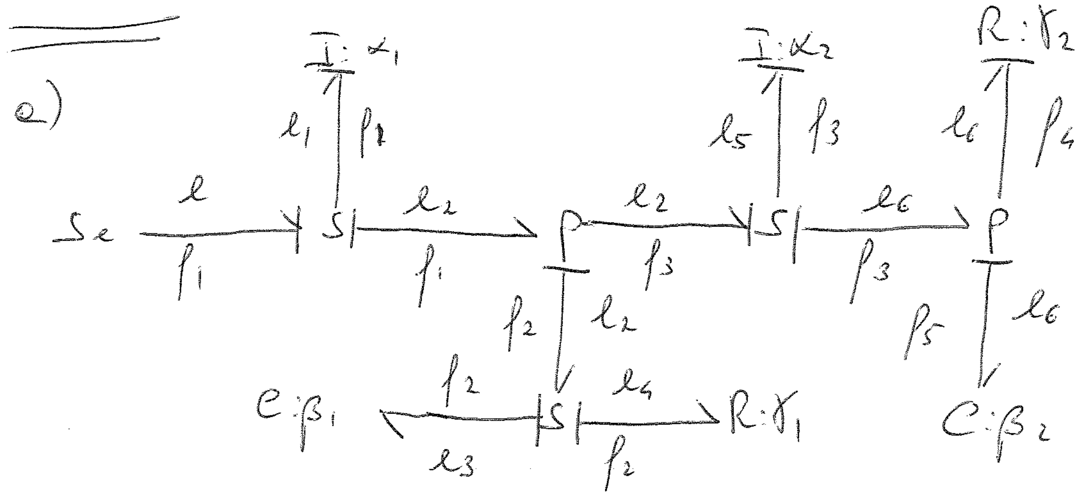
Exercise 1

a)



b) State variables:

$$x_1 = f_1, \quad x_2 = e_3, \quad x_3 = f_3, \quad x_4 = e_6$$

By ~~By~~ ~~T~~the relationships between efforts and flow of the dynamic components ~~xxxx~~ and the constraints imposed by the 's' and 'p' junctions, we have:

$$\begin{cases} \dot{x}_1 = \frac{1}{\alpha_1} e_1 \\ \dot{x}_2 = \frac{1}{\beta_1} f_2 \\ \dot{x}_3 = \frac{1}{\alpha_2} e_5 \\ \dot{x}_4 = \frac{1}{\beta_2} f_5 \end{cases} \quad \text{with} \quad \begin{cases} e_1 = e - e_2 = e - e_3 - e_4 = \\ \quad = e - x_2 - \gamma_1 f_2 = \\ \quad = e - x_2 - \gamma_1 (x_1 - x_3) \\ f_2 = f_1 - f_3 = x_1 - x_3 \\ e_5 = e_2 - e_6 = e_3 + e_4 - x_4 = \\ \quad = x_2 + \gamma_1 (x_1 - x_3) - x_4 \\ f_5 = x_3 - \frac{x_4}{\gamma_2} \end{cases}$$

The equations of the model in the state space are:

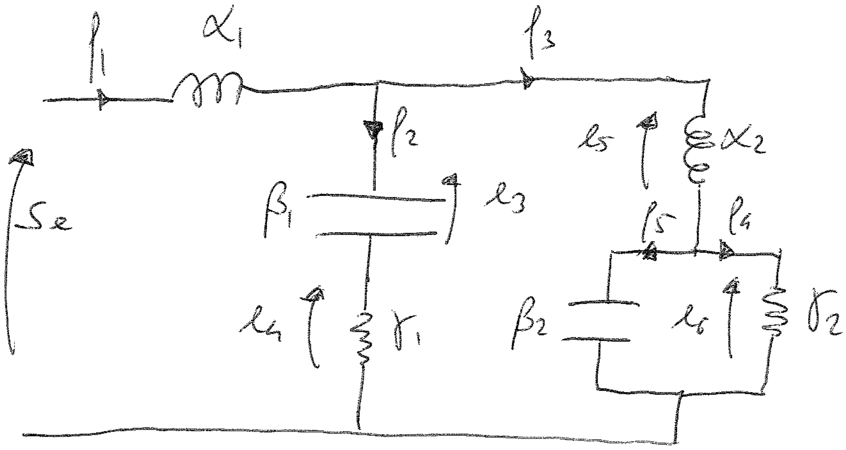
$$\dot{x}_1 = -\frac{\gamma_1}{\alpha_1} x_1 - \frac{1}{\alpha_1} x_2 + \frac{\gamma_1}{\alpha_1} x_3 + \frac{1}{\alpha_1} e$$

$$\dot{x}_2 = \frac{1}{\beta_1} x_1 - \frac{1}{\beta_1} x_3$$

$$\dot{x}_3 = \frac{\gamma_1}{\alpha_2} x_1 + \frac{1}{\alpha_2} x_2 - \frac{\gamma_1}{\alpha_2} x_3 - \frac{1}{\alpha_2} x_4$$

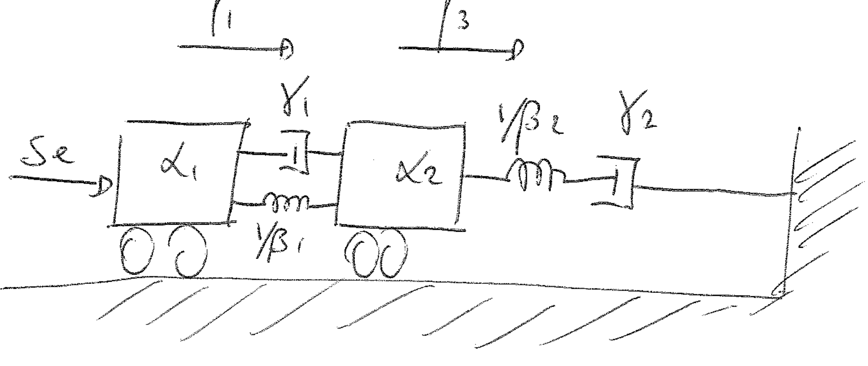
$$\dot{x}_4 = \frac{1}{\beta_2} x_3 - \frac{1}{\beta_2 \gamma_2} x_4$$

c)



f_1
α_1
f_3
f_2
e_5
α_2
Se
β_1
e_3
f_5
f_4
e_4
γ_1
β_2
e_6
γ_2

d)



f_1
f_3
γ_1
$1/\beta_2$
γ_2
Se
α_1
α_2
$1/\beta_1$

Exercise 2

The input-output equation of the model can be written as follows:

$$y(t) - 0.2q^{-1}y(t) = u(t) - 0.1q^{-1}u(t)$$

$$\frac{y(t)}{u(t)} = \frac{1-0.1q^{-1}}{1-0.2q^{-1}}$$

The discrete time transfer function is:

$$G(z) = \frac{1-0.1z^{-1}}{1-0.2z^{-1}} \quad \text{with } Y(z) = G(z)U(z)$$

with $Y(z)$ and $U(z)$ the z-transforms of the output and input signals, respectively.

From $G(z)$ the frequency response function is ~~[illegible]~~ computed as:

$$G(i\omega) = G(z)\big|_{z=e^{i\omega T}} \quad \text{with } T \text{ the sampling time.}$$

a) $\Phi_y(\omega) = |G(i\omega)|^2 \Phi_u(\omega) = |G(i\omega)|^2 \lambda_u$

since $u(t)$ is white noise

$$|G(i\omega)|^2 = G(i\omega)\overline{G(i\omega)} = \frac{1-0.1e^{-i\omega T}}{1-0.2e^{-i\omega T}} \cdot \frac{1-0.1e^{i\omega T}}{1-0.2e^{i\omega T}} =$$

$$= \frac{1.01 - 0.2\cos\omega T}{1.04 - 0.4\cos\omega T} \quad \text{by Euler formulas}$$

b) $\Phi_{yu}(\omega) = \dfrac{1-0.1e^{-i\omega T}}{1-0.2e^{-i\omega T}}\lambda_u$

Exercise 3

a) The modelica code implements a discrete time system (digital filter) described by the following input - output equation in the time domain

$$y(t) = y(t-1) + 2y(t-2) + 3y(t-3) + u(t-1) + 2u(t-2)$$

provided that the vectors y_prev and u_prev ~~are~~ filled with the previous samples of the output and input signals, respectively. I.e.,

$$y\_prev[i] = y(t-i)$$

b) The system in figure is described by the following model:

$$\begin{cases} \dot{p}_1 = v_1 \\ \dot{p}_2 = v_2 \\ m\dot{v}_1 = F - K(p_1-p_2) - d(v_1-v_2) \\ 2m\dot{v}_2 = K(p_1-p_2) + d(v_1-v_2) \end{cases}$$

where pi and vi are the positions and velocities of the two masses.

The model can be simulated through the following Modelica code:

model train

input Real F, "fora";

output Real x[4], "state_vector";

Real p_1, "position1";

Real p_2, "position2";

Real v_1, "velocity1";

Real v_2, "velocity2";

parameter m=1, "mass";

parameter K=1, "spring";

parameter d=1, "damping";

equation

$der(p_1) = v_1;$

$der(p_2) = v_2;$

$$der(v_1) = \frac{1}{m}\left[F - K(p_1 - p_2) - d(v_1 - v_2)\right];$$

$$der(v_2) = \frac{1}{2m}\left[K(p_1 - p_2) + d(v_1 - v_2)\right];$$

$x[1] = p_1;$

$x[2] = p_2;$

$x[3] = v_1;$

$x[4] = v_2;$

end train

Exercise 4

a) Denote by θ_0 the real vector of parameters and $\hat{\theta}$ the estimate, i.e, the solution of the system identification problem obtained through the PE method. We have:

$$E[(\hat{\theta}-\theta_0)(\hat{\theta}-\theta_0)^T] \propto \frac{1}{N} \cdot \lambda$$

b) The bias error in the parameter estimate is induced by deficiencies in the model Structure. That is, the chosen model is not able to describe the system.