## Chalmers University of Technology Department of Signals and Systems

## **ESS101** Modelling and simulation

Examination date 091020

*Time:* 14.00 – 18.00

*Teacher:* Paolo Falcone, 772 1803

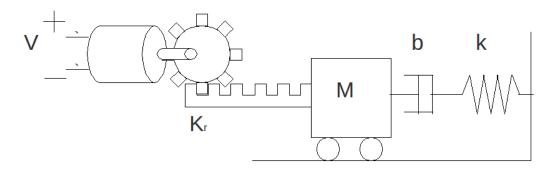
Allowed material during the exam: Mathematics Handbook.

The exam consists of 5 exercises of a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Results are announced on the notice board at the latest Oct 30. You can discuss with teacher and TAs the grading of your exam on Nov 2 at 12.30-13.15 at the Department of Signals and Systems.

Exercise 1 (5 p)

Consider the system in the figure below, consisting (from the left) of a DC motor, a mechanism for converting the rotating into a translatory motion, a mass, a friction and elastic elements.



- (a) Draw a bond graph of the system and mark the causality. (2p)
- (b) Derive a state-space model from the obtained bond graph. (3p)

Exercise 2 
$$(5 p)$$

(a) The parameters a and b in the system

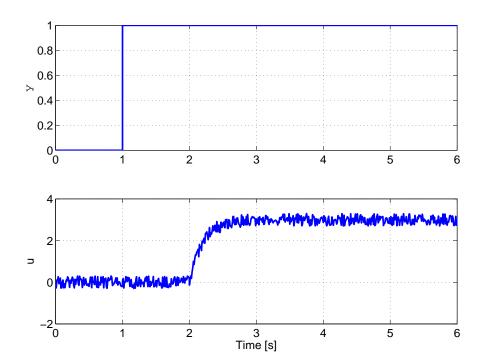
$$y(t) + ay(t-1) = bu(t-1) + v(t)$$

have to be estimated. The system is controlled by a proportional controller  $u(t) = -k_p(r(t) - y(t))$ , where r(t) is a reference signal. Can the parameters a and b be uniquely determined by any identification method for every r(t)? Motivate the answer and describe the procedure for identifying the parameters. (2p)

(b) Consider the input output data in the figure below. By using the transient analysis method, determine a mathematical model of the system. (3p)

Exercise 
$$3$$
 (5 p)

A stationary time continuous stochastic process  $\{w(t)\}$  has spectra  $\Phi_w(\omega)$ . Describe  $\{w(t)\}$  as output signal from a linear system driven by white noise  $\{e(t)\}$  with variance  $\lambda_e=1$ , when



(a) 
$$\Phi_w(\omega) = \frac{2}{\omega^2 + 2}$$

(b) 
$$\Phi_w(\omega) = \frac{1}{(\omega^2+2)(\omega^2+1)}$$

Consider the following system

$$y(t) = \frac{0.0907q^{-1} + 0.1070q^{-2}}{1 - 1.397q^{-1} + 0.5918q^{-2}}u(t) + \frac{1 - 1.438q^{-1} + 0.599q^{-2}}{1 - 1.397q^{-1} + 0.5918q^{-2}}e(t)$$

- (a) Classify the system, i.e., system type and number of parameters. (1p)
- **(b)** Calculate the predictor. (3p)
- (b) Show that the predicted output is function only of past samples of the input and output signals. (1p)

Exercise 5 (5 p)

Simulate for five steps the following system with both Forward and Backward Euler methods.

 $\dot{x}(t) = \begin{bmatrix} -2 & 2\\ 1 & 0 \end{bmatrix} x(t),$ 

with  $x(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ . Motivate the choice of the integration step in both cases.