Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Examination date 090818

Time: 14.00 – 18.00

Teacher: Paolo Falcone, 772 1803

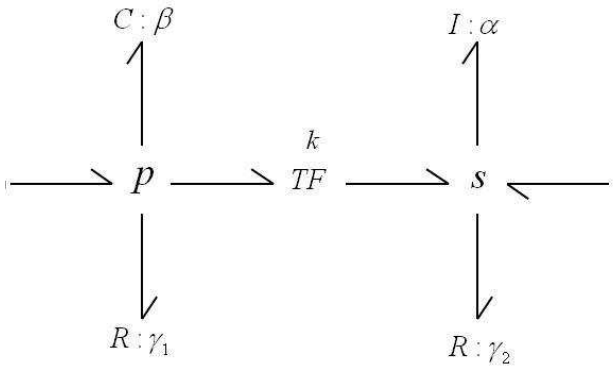
Allowed material during the exam: Mathematics Handbook.

The exam consists of 5 exercises of a total of 25 points. Nominal grading is according to 12/17/21 points. You need 12 points to pass the exam with grade 3, 17 points to pass with grade 4 and 21 to pass with grade 5. Solutions and answers should be written in English, unambiguous and well motivated, but preferably short and concise.

Results are announced on the notice board at the latest Sept 1. You can check the grading of your exam on Sept 3 at 12.30-13.15 at the Department of Signals and Systems.

Exercise 1 (5 p)

Consider the bond graph in the figure below.



(a) Show all the possible combinations of left and right inputs subject to the causality of the graph. (2p)

(b) Mark the causality of the obtained graphs. (1p)

(c) For one of the obtained system, formulate a state space model. (2p)

Exercise 2 (5 p)

Consider the second order system

$$\dot{x}_1 = x_2 + u$$
$$\dot{x}_2 = (1 - x_1^2)x_2 - x_1$$

(a) Determine an equilibrium point and linearize the system around that equilibrium point. (2p)

(b) Sketch a Simulink block scheme for simulating the linearized model. (3p)

Exercise 3 (5 p)

Consider the system

$$y(t) = u(t-1) + 0.1u(t-2) + e(t)$$

where $\{e(t)\}$ is white noise with variance 1.
What is the value of the estimated parameters when the number of observations approaches infinity and

(a) $\{u(t)\}$ is white noise with variance 1.

(b) $\{u(t)\}$ has covariance function

$$
\begin{aligned}
R_u(0) &= 1, \\
R_u(1) &= 0.5, \\
R_u(2) &= 0.25, \\
R_u(3) &= 0.125, \\
&\text{and so on.}
\end{aligned}
$$

The input signal can be assumed to be independent of the disturbance.

Exercise 4 (5 p)

(a) Provide an example of stable dynamical system and integration step, such that the *forward* Euler method is stable. (1p)

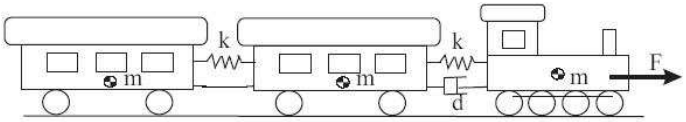
(b) Provide an example of stable dynamical system and integration step, such that the *backward* Euler method is unstable. (1p)

(b) Simulate the following system with the backward Euler method for three steps and initial condition $x(0) = [0\ 3]^T$. (3p)

$$\dot{x}(t) = \begin{bmatrix} -2 & 2 \\ 0 & -1 \end{bmatrix} x(t)$$

Exercise 5 (5 p)

Derive a state space model for the system sketched below. Let the force F be



the input signal and the velocity of the last wagon the output signal.