SOLUTIONS OF THE EXAM
ESS101 - MODELING AND SIMULATION

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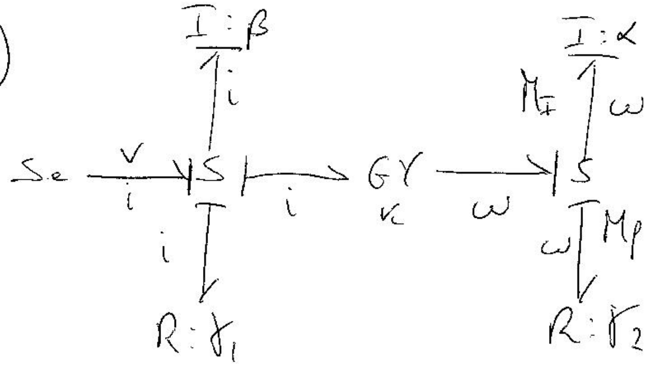
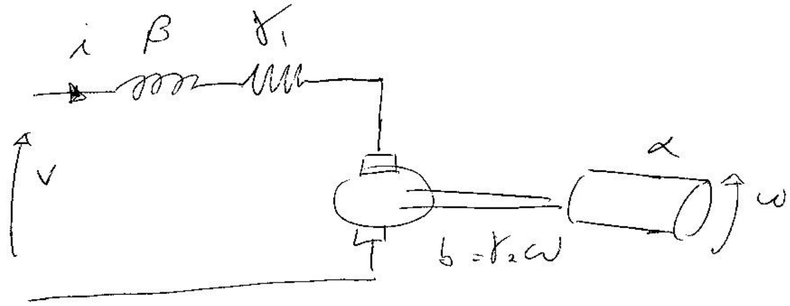
EXERCISE 1

a)



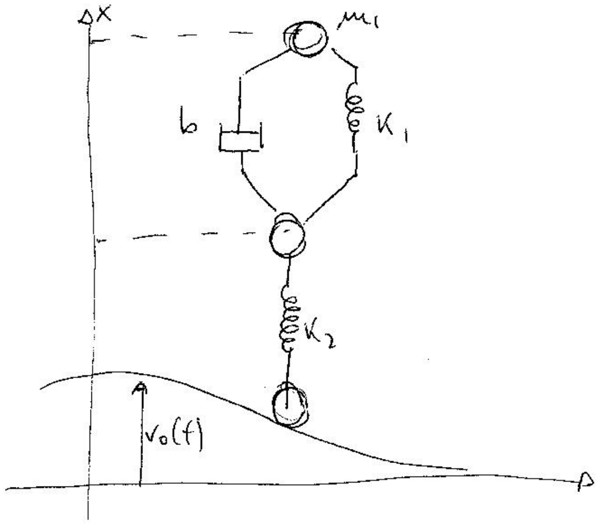
b)



c)

$$\begin{cases} v = \gamma_1 i(t) + \beta \dot{i}(t) + K\omega(t) \\ \alpha \dot{\omega}(t) = K i(t) - \gamma_2 \omega(t) \end{cases}$$

EXERCISE 2



x_2 velocity of mass m_1

x_4 " " m_2

a)

$$\dot{x}_1 = x_2 - x_4$$

$$\dot{x}_2 = -\frac{k_1}{m_1}x_1 - \frac{b}{m_1}x_2 + \frac{b}{m_1}x_4 - g$$

$$\dot{x}_3 = x_4 - \dot{v}_0(t)$$

$$x_4 = \frac{k_1}{m_2}x_1 + \frac{b}{m_2}x_2 - \frac{k_2}{m_2}x_3 - \frac{b}{m_2}x_4 - g$$

$$y = x_2$$

b)

$$\dot{x}_1 = x_2 - \dot{v}_0$$

$$x_2 = -\frac{k_1}{m_1}x_1 - \frac{b}{m_1}x_2 + \frac{b}{m}\dot{v}_0(t) - g$$

$$y = x_2$$

c)

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -\frac{k_1}{m_1} & -\frac{b}{m_1} \end{pmatrix} x + \begin{pmatrix} -1 \\ \frac{b}{m_1} \end{pmatrix} \dot{v}_0(t) - \begin{pmatrix} 0 \\ 1 \end{pmatrix} g$$

$$y = [0 \quad 1]\, x$$

$$\therefore G(s) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s & -1 \\ \frac{k_1}{m_1} & s+\frac{b}{m_1} \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ \frac{b}{m_1} \end{pmatrix} =$$

$$= \frac{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} s+\frac{b}{m_1} & 1 \\ -\frac{k_1}{m_1} & s \end{pmatrix} \begin{pmatrix} -1 \\ \frac{b}{m_1} \end{pmatrix}}{s\left(s+\frac{b}{m_1}\right)+\frac{k_1}{m_1}} =$$

$$= \frac{sb + k_1}{s(m_1 s + b) + k_1}$$

$$\Phi_{v_1(t)}(\omega) = |G(j\omega)|^2 \lambda^2$$

EXERCISE 3

a) $y(t) = 0.5\, u(t-1) + \frac{1}{1+\alpha z^{-1}} \xi(t)$

$$y(t) = -\alpha y(t-1) + 0.5\, u(t-1) + 0.5\,\alpha\, u(t-2) + \xi(t)$$

$$y(t) - 0.5\, u(t-1) = \alpha\left[-y(t-1) + 0.5\, u(t-2)\right] + \xi(t)$$

Let's define:

$$\left.\begin{array}{l} \tilde{y}(t) = y(t) - 0.5\, u(t-1) \\ \tilde{\varphi}(t) = -y(t-1) + 0.5\, u(t-2) \\ \tilde{\theta} = \alpha \end{array}\right\} \Rightarrow \underline{\tilde{y}(t) = \tilde{\theta}^T \tilde{\varphi}(t) + \xi(t)}$$

The least squares method can be applied on the tilded version.

b) $y(t) = a\, y(t-1)^2 + b_1 u(t-1) + b_2 u(t-2)^3 + \xi(t)$

The system can be rewritten as:

$y(t) = \theta^T \varphi(t)$ with:

$$\theta = \begin{bmatrix} a \\ b_1 \\ b_2 \end{bmatrix} \quad \varphi(t) = \begin{bmatrix} y(t-1)^2 \\ u(t-1) \\ u(t-2)^3 \end{bmatrix}$$

EXERCISE 4

a) Consider the ~~syst~~ model:

$$\dot{x} = Ax$$

Assume A is a negative scalar. In the case A is a matrix, it can be diagonalized and the scalar case can be considered.

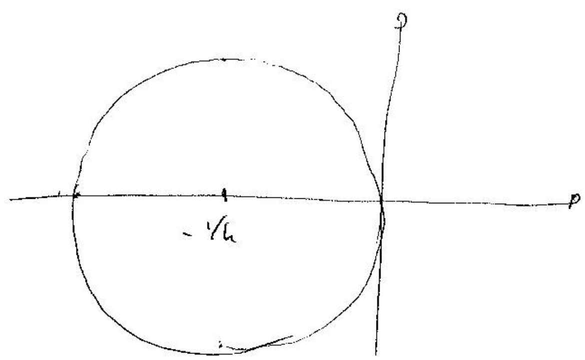
By applying the forward Euler method:

$$x_{k+1} = (1 + Ah)\, x_k$$

Said discrete time model is stable if

$$|1 + Ah| < 1$$

For some positive h ~~the poles of the~~ poles of the continuous time system lying within the circle with radius $1/h$ centered in $-1/h$ are mapped into stable poles of the d.t. system.



b) For the backward Euler method

$x_k = (1 - hA)^{-1} x_k \Rightarrow$ The method is always stable.

b) The system has two poles $\lambda_1 = -1$, $\lambda_2 = -2$

In the FE method $h \leq 1$