

ESS101 Modelling and simulation
Examination date 081021

Time: 14.00 – 18.00

Teacher: Paolo Falcone, 772 1803

Allowed material during the exam: Mathematics Handbook.

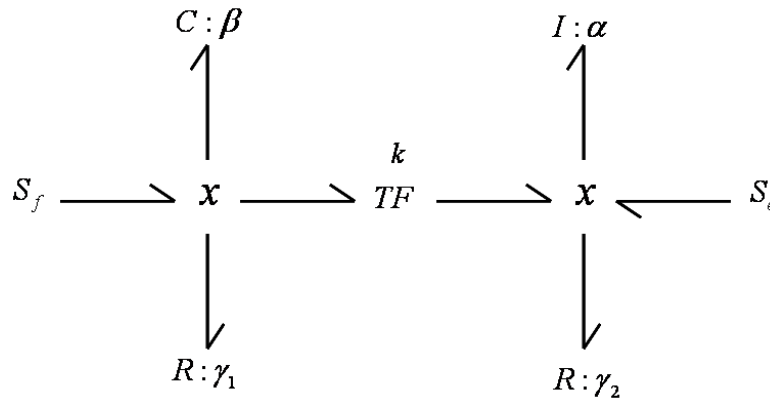
The exam consists of 5 exercises of a total of 25 points. Nominal grading according to 12/17/21 points, you need 12 points to pass the course with grade 3, 17 points to pass with grade 4 and 21 to pass the course with grade 5. Solutions and answers should be written in English and be unambiguous and well motivated, but preferably short and concise.

Results are announced on the notice board at the latest Nov 4. You can check the grading of your exam on Nov 5 at 12.30-13.15 at the Department of Signals and Systems.

Exercise 1

(5 p)

Consider the bond graph in the figure below.



(a) Specify the two junctions 'x' in order to have a conflict-free graph and mark the causality. Motivate the answer. (2p)

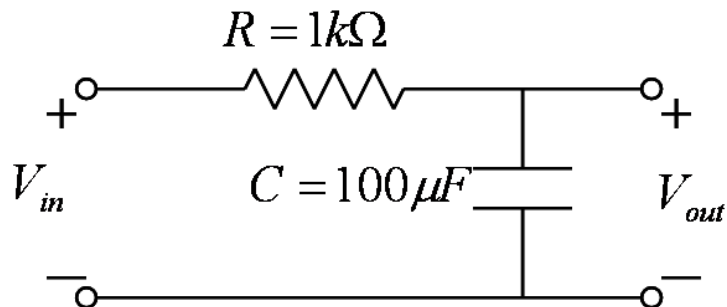
(b) Derive a state space model from the obtained graph. (2p)

(c) How does the answer to a) change if the flow source on the left is replaced by an effort source? (1p)

Exercise 2

(5 p)

Consider the RC filter in the figure below.



- (a) Compute the spectrum of the signal V_{out} , when V_{in} is a white noise with variance 1. (3p)
- (b) Sketch a diagram of the spectrum of V_{out} versus the frequency. (1p)
- (c) How does the diagram at point b) change when C decreases? Motivate the answer. (1p)

Exercise 3 (5 p)

A system is given as

$$y(t) = u(t - 1) + 0.5u(t - 2) + e(t)$$

where $\{e(t)\}$ is white noise with variance 1. Assume an $ARX(1, 1)$ model is used to identify the system.

What is the value of the estimated parameters when the number of observations approaches infinity and

- (a) $\{u(t)\}$ is white noise with variance 1.
- (b) $\{u(t)\}$ has covariance function

$$\begin{aligned} R_u(0) &= 1, \\ R_u(1) &= 0.5, \\ R_u(2) &= 0.25, \\ R_u(3) &= 0.125, \\ &\text{and so on.} \end{aligned}$$

The input signal can be assumed to be independent of the disturbance.

Exercise 4 (5 p)

- (a) What is the (differentiation) index of the following DAE? (2p)

$$\begin{aligned} \dot{x}_1 &= -x_1 + x_2x_1 \\ \dot{x}_2 &= -x_1^2 + x_2x_3 \\ 0 &= x_1^2 + x_2 + x_3^2. \end{aligned}$$

Motivate the answer.

(b) Show an example of nonlinear DAE with index 2. (2p)

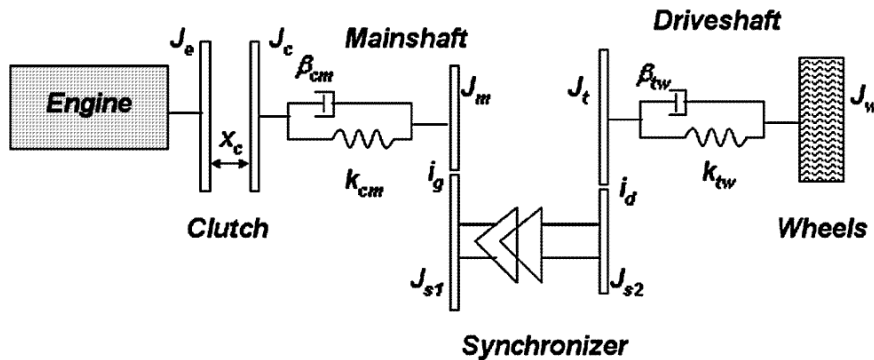
(c) What is the index of the following linear DAE? (1p)

$$\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 12 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 12 & 6 & 0 & 0 \\ 18 & 18 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Suggestion. The index can be computed by using well-known theoretical results.

Exercise 5 (5 p)

A vehicle driveline can be described according to the figure below, where J_e is the engine inertia, x_c is the clutch position, J_c is the clutch inertia, β_{cm} and k_{cm} are the damping and the stiffness coefficients, respectively, of the axle connecting the main shaft and the clutch, J_m is the inertia of the main shaft, i_g and i_d are the gear and the differential ratios, respectively, J_{s1} and J_{s2} are the inertias of the two disks connected to the synchronizer, J_t , β_{tw} and k_{tw} are the inertia, the damping and the stiffness coefficients of the drive shaft, respectively, and J_w is the wheel inertia. The two inertias connected to the synchronizer have the same angular speed.



Assume a rigid main shaft (i.e., the clutch position and speed equal to the main shaft position and speed, respectively) and that the transmitted torque to the clutch T_c is a known function of the clutch position x_c , i.e., $T_c = T_c(x_c)$.

Determine a state space representation for the driveline. Let the engine torque, the clutch position x_c be the input signals, the wheel (vehicle) longitudinal velocity be the output and the load to the wheel an external disturbance.

Help. In the state equation describing the clutch speed dynamics, the inertia is equal to $J_c + J_m + \frac{1}{i_g^2} \left(J_{s1} + J_{s2} + \frac{J_t}{i_d^2} \right)$.