SOLUTIONS OF THE ESS101 - MODELING AND SIMULATION EXAM

Examination date 21/10/08

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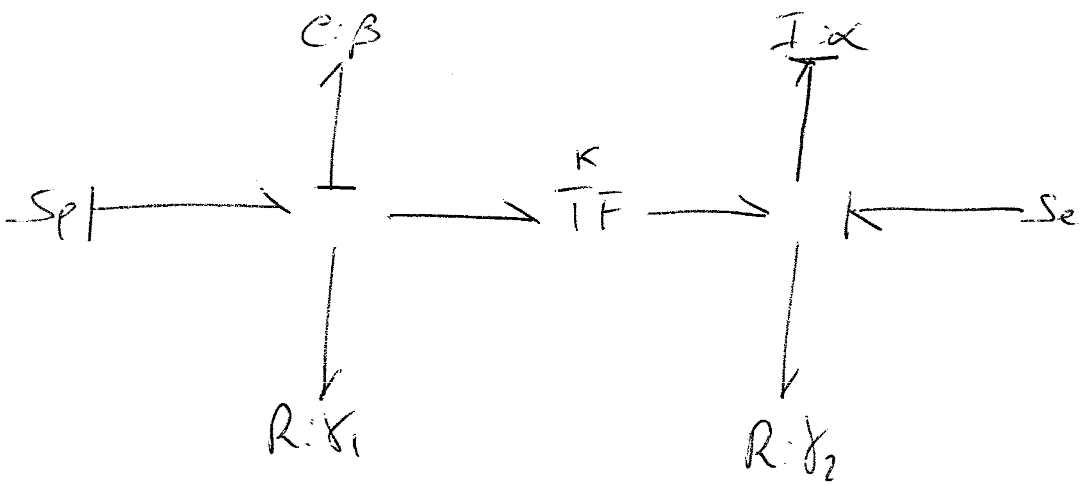
Exercise 1

a)



Step 1

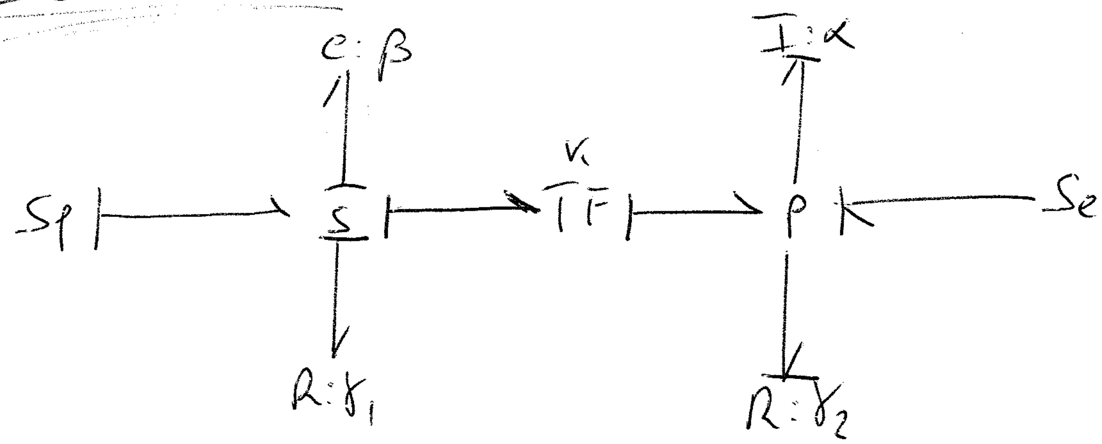
Mark the causality of Sf, Se, C and I elements.

Step 2

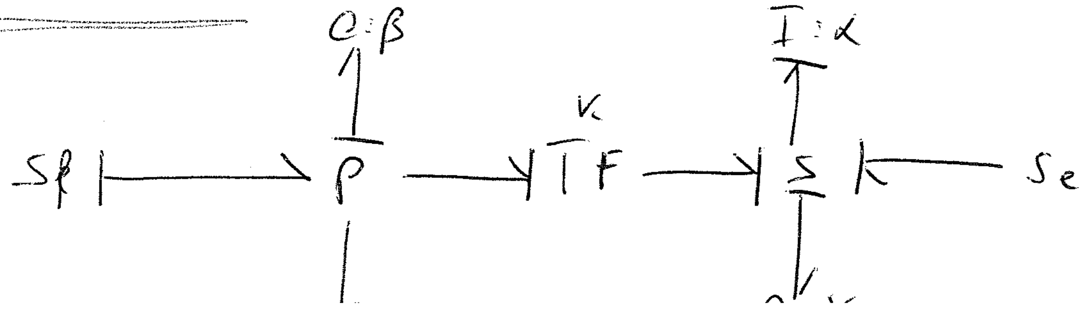
Check the causality of the graph ~~[illegible]~~ for the two possible causalities of the transformer

There are two admissible solutions:

SOLUTION 1

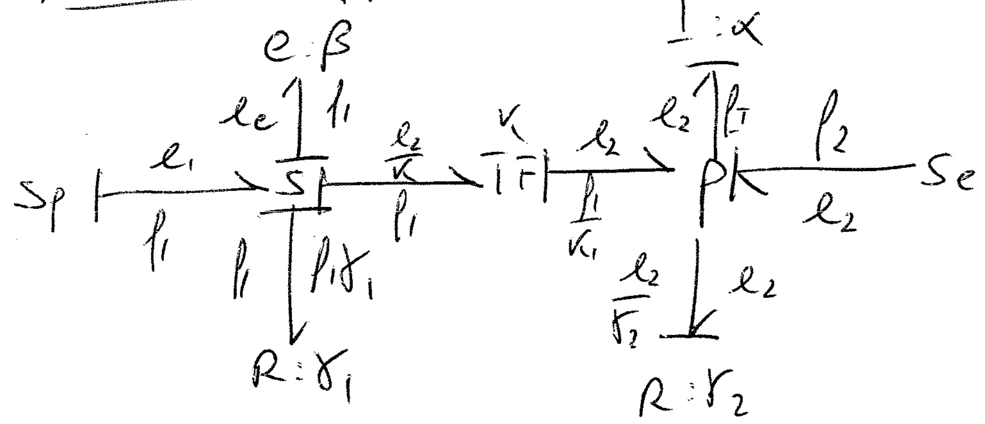


SOLUTION 2



b) Let's derive a state space model for the

SOLUTION 1:

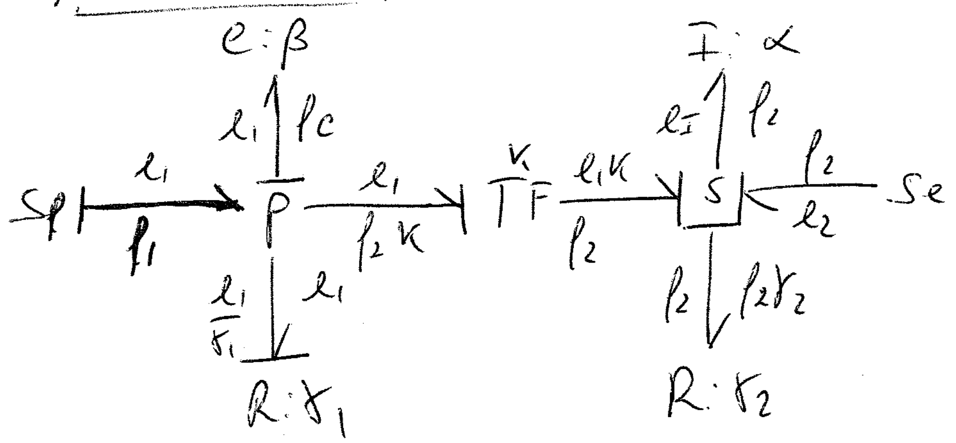


Denote by $x = \begin{bmatrix} e_c \\ f_I \end{bmatrix}$ and $u = \begin{bmatrix} f_1 \\ e_2 \end{bmatrix}$ the state and input vectors. We have

$$\dot{x} = \begin{bmatrix} 1/\beta & 0 \\ 0 & 1/\alpha \end{bmatrix} u$$

Let's derive a state space model for the

SOLUTION 2:

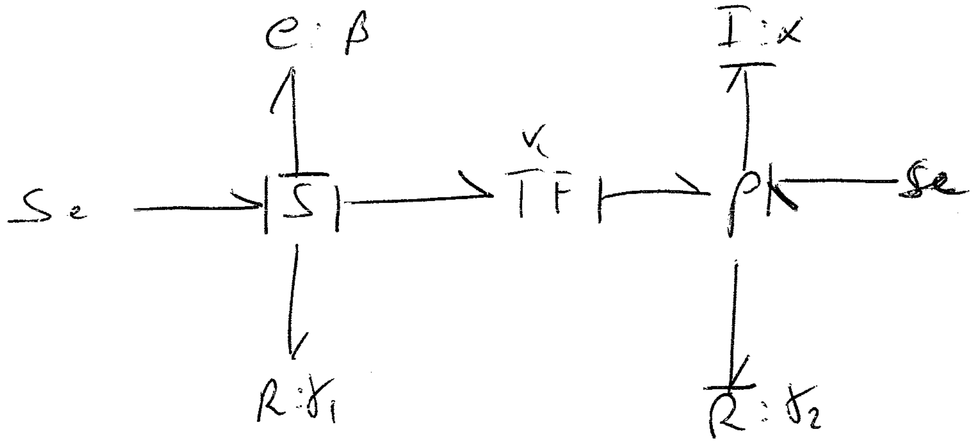


Denote by $x = \begin{bmatrix} e_1 \\ f_2 \end{bmatrix}$ and $u = \begin{bmatrix} f_1 \\ e_2 \end{bmatrix}$.

the State space model is:

$$\dot{x} = \begin{bmatrix} -\frac{1}{\beta\gamma_1} & -\frac{\kappa}{\beta} \\ +\frac{\kappa}{\alpha} & -\frac{\gamma_2}{\alpha} \end{bmatrix} x + \cancel{\begin{bmatrix} \frac{1}{\beta} \\ \frac{1}{\alpha} \end{bmatrix} u} \begin{bmatrix} \frac{1}{\beta} & \\ & \frac{1}{\alpha} \end{bmatrix} u$$

e) If the flow source on the left is replaced by an effort flow, the following graph is obtained:



Then we have a unique solution that is conflict free.

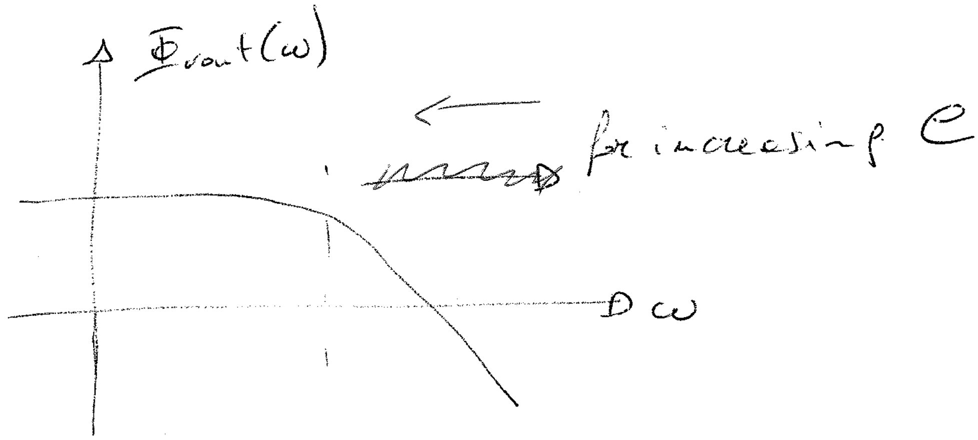
Exercise 2

a) The spectrum of Vout is:

$\Phi_{vout}(\omega) = |G(j\omega)|^2 \, \Phi_{vin}(\omega)$ where:

$G(s) = \frac{1}{1+s\tau}$ with $\tau = RC$ and $\Phi_{vin}(\omega) = 1 \ \forall \omega$

c, b) The spectrum of Vout looks like:



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Exercise 3

The predictor is:

$$\hat{y}(t,\theta) = \theta^T \varphi(t) \quad \text{with} \quad \theta = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}, \quad \varphi(t) = \begin{bmatrix} y(t-1) \\ u(t-1) \end{bmatrix}$$

Appling the least squares formula:

$$\hat{\theta}_N = \begin{pmatrix} \frac{1}{N}\sum_{t=1}^{N} y^2(t-1) & \frac{1}{N}\sum_{t=1}^{N} y(t-1)u(t-1) \\ \frac{1}{N}\sum_{t=1}^{N} y(t-1)u(t-1) & \frac{1}{N}\sum_{t=1}^{N} u^2(t-1) \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{N}\sum_{t=1}^{N} y(t-1)y(t) \\ \frac{1}{N}\sum_{t=1}^{N} u(t-1)y(t) \end{pmatrix}$$

As $N \to \infty$

$$\hat{\theta}_N \to \begin{pmatrix} R_y(0) & R_{yu}(0) \\ R_{yu}(0) & R_u(0) \end{pmatrix}^{-1} \begin{pmatrix} R_y(1) \\ R_{yu}(1) \end{pmatrix}$$

$$R_y(0) = \bar{E}[y(t)y(t)] = \bar{E}[u^2(t-1) + 0.5u^2(t-2) + e^2(t) + 2u(t-1)u(t-2) + 2u(t-1)e(t) + 2u(t-2)e(t)]$$

$$R_{yu}(0) = \bar{E}[y(t)u(t)] = \bar{E}[u(t-1)u(t) + 0.5u(t)u(t-2) + u(t)e(t)]$$

$$R_y(1) = \bar{E}[y(t)y(t-1)] = \bar{E}[u(t-1)y(t-1) + 0.5u(t-2)y(t-1) + y(t-1)e(t)] =$$

$$= R_{uy}(0) + 0.5R_{uy}(1)$$

$$R_{yu}(1) = \bar{E}[y(t)u(t-1)] = \bar{E}[u^2(t-1) + 0.5u(t-1)u(t-2) + e(t)u(t-1)] =$$

$$= R_u(0) + 0.5R_u(1)$$

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a) $R_y(0) = 2.5$ $R_u(0) = 1$

$R_{yu}(0) = 0$

$R_{yu}(1) = 1$

$R_y(1) = 0.5$

$$\hat{\theta}_N = \begin{pmatrix} 2.5 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.4 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 1 \end{pmatrix}$$

b) $R_y(0) = 3\cancel{.2}5$ $3{,}5$

$R_u(0) = 1$

$R_{yu}(0) = 0.625$

$R_{yu}(1) = 1.25$

$R_y(1) = 1.25$

$$\hat{\theta}_N = \begin{pmatrix} \overset{3{,}5}{3\cancel{.2}5} & 0.625 \\ 0.625 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1.25 \\ 1.25 \end{pmatrix} = \frac{1}{3.25 - 0.390} \begin{pmatrix} 1 & -0.625 \\ -0.625 & 3.25 \end{pmatrix} \begin{pmatrix} 1.25 \\ 1.25 \end{pmatrix}$$

$$= \begin{pmatrix} 0.1639 \\ 1.1475 \end{pmatrix} \qquad \begin{pmatrix} 0{,}1508 \\ 1{,}1558 \end{pmatrix}$$

Exercise 5

a) The DAE is of type:

$$\dot{x} = f(x,y) \quad \text{with} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{and } y = x_3$$
$$0 = g(x,y)$$

Moreover we observe, $g_y(x,y) = 2x_3 \neq 0 \quad \forall x_3 \neq 0$

The index of the DAE is 1 ~~for~~ $\forall x \in R^3 - \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : y = 0 \right\}$

b) The following DAE

$$\dot{x}_1 = -x_1 + x_2^2 \qquad \text{has index } 2 \ \forall x \in R^2 - \{\bar{0}\}$$
$$0 = x_1^2 + 5$$

3) It is straightforward to recognize that

$$\det(\lambda E + F) \neq 0$$

Moreover, be $U = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$; $V = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

The following equalities hold:

$$U \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} V \quad \text{and} \quad U \begin{bmatrix} 2 & 3 & 0 & 0 \\ 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} V$$

Then $N = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$. Since $N^2 = 0$ we deduce the index is 2.

Exercise 5

$$J_e \dot{\omega}_e = \underset{e}{T_e} - \underset{T_c}{T_e(x_e)}$$

$$\left[\cancel{J_e} + J_{eq}(i_g i_d)\right]\dot{\omega}_e = T_e(x_e) - \frac{1}{i_g i_d}\left[K_{Tw}\Delta\theta_{ew} + \beta_{tw}\left(\frac{\omega_e}{i_g i_d} - \omega_w\right)\right]$$

$$J_w \dot{\omega}_w = K_{tw}\Delta\theta_{ew} + \beta_{tw}\left(\frac{\omega_e}{i_g i_d} - \omega_w\right) - T_L(\omega_w)$$

$$\Delta\dot{\theta}_{ew} = \frac{\omega_e}{i_g i_d} - \omega_w$$

with:

$$J_{eq}(i_g, i_d) = J_e + J_m + \frac{1}{i_g^2}\left(J_{s_1} + J_{s_2} + \frac{J_t}{i_d^2}\right)$$