Chalmers University of Technology
Department of Signals and Systems

ESS101 Modelling and simulation

Examination date 080819

Time: 14.00 – 18.00

Teacher: Paolo Falcone, tel 031-772 1803.

Allowed material during the exam: Mathematics Handbook.

The exam consists of 5 exercises of a total of 25 points. Nominal grading according to 12/17/21 points, you need 12 points to pass the course with grade 3, 17 points to pass with grade 4 and 21 to pass the course with grade 5. Solutions and answers should be written in English and be unambiguous and well motivated, but preferably short and concise.

Results are announced on the notice board at the latest Sept 3. You may check your grading of your exam on Sept 3 at 12.15-13.15 at the Department of Signals and Systems.

GOOD LUCK!

Exercise 1 (5 p)

(a) In vehicle dynamics and tire modelling *slip* is a commonly used concept. Slip is defined as

$$slip = \frac{v_{fordon} - v_{hjul}}{v_{fordon}}$$

i.e. the relative velocity difference between wheel and vehicle. What problem can arise when using this in simulation? (1p)

(b) What is the index of the following DAE system?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 2 \end{bmatrix}$$

(2p)

(c) Let $\{w(t)\}$ be a stationary stochastic process. $w(t)$ is generated from the time discrete relation

$$w(t) - 0.4w(t-1) = e(t) - 0.5e(t-1)$$

where $\{e(t)\}$ is white noise with variance λ_e. Determine the spectrum $\Phi(\omega)$ for $\{w(t)\}$. (2p)

Exercise 2 (5 p)

We would like to use identification and the prediction error method to estimate the parameters in the model

$$y(t) + ay(t-1) = bu(t-1) + e(t)$$

where $e(t)$ is white noise. The true system is

$$y(t) = 0.6u(t-1) + 0.3u(t-2) + v(t)$$

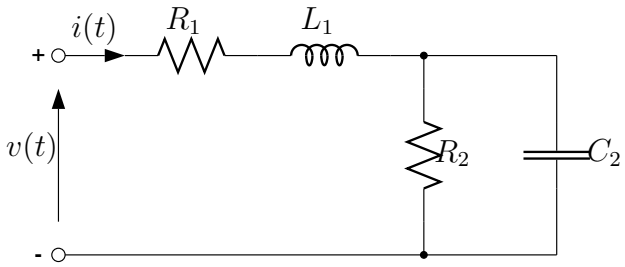
where the input, $u(t)$, is white noise with variance 1 and the disturbance, $v(t)$, is white noise with variance 2. v and u are assumed to be uncorrelated. To which

values does the estimation of a and b converge, when the number of samples N is large ($N \to \infty$)?
Hint: Least squares estimation $\hat{\theta}_N$ can be calculated using

$$\hat{\theta}_N = (\frac{1}{N}\sum_{t=1}^{N}\varphi(t)\varphi^T(t))^{-1}(\frac{1}{N}\sum_{t=1}^{N}\varphi(t)y(t))$$

Exercise 3 (5 p)

Consider the electric circuit below.



(a) Draw a bond graph and mark causality for the electric circuit. Should $v(t)$ or $i(t)$ be chosen as input signal? Motivate! (3p)

(b) Sketch a mechanical system which has the same bond graph as the electrical system. (2p)

Exercise 4 (5 p)

You are supposed to develop a flash light that has no batteries. Instead the energy needed to light it is generated by cranking it, similar to a bicycle dynamo.

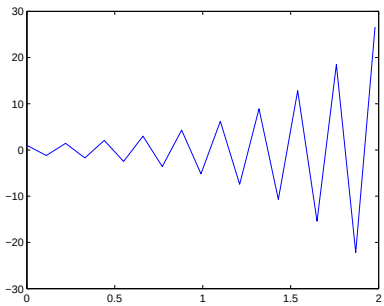
Develop a mathematical model for the flash light. The flash light consists of the cranking device (a handle and a reduction gear), the dynamo (a generator - an electric motor run backwards), the energy storage (a capacitor) and the light bulb (a resistor). The capacitor and the resistor is connected in parallel.

Exercise 5 (5 p)

A simulation of the system

$$\dot{x}(t) = -20x(t), x(0) = 1$$

using Euler's method, with a step size of 0.11 seconds, gives the following result:



(a) Explain and verify the result. (3p)

(b) Suggest two different ways to improve the result! (2p)