SOLUTIONS OF THE ESS101 - MODELING AND SIMULATIONS EXAM

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Exercise 1

a) A division by 0 will occur if $V_{pordon} = 0$

b) The considered DAE system is in the form:

$$E\dot{x} + Fx = G$$

where the couple (E, F) form a regular matrix pencil.
Then two nonsingular matrices exist U and V such that

$$UEV = \begin{bmatrix} I_2 & O_2 \\ O_2 & N \end{bmatrix} \quad \text{and} \quad UFV = \begin{bmatrix} C & O_2 \\ O_2 & I_2 \end{bmatrix}$$

where I_2 is the identity matrix of dimension 2, O_2 is a 2x2 matrix of zeroes, $N \in R^2$ and $C \in R^2$ in our case.

Indeed $U = I_4$ and $V = I_4$ in our case. ~~Then~~

Since $N^2 = 0$ we conclude that the index of the system is 2.

c) The considered system can be rewritten as follows:

$$w(t) = G(q)\, e(t), \quad \text{where} \quad G(q) = \frac{1 - 0.5 q^{-1}}{1 - 0.4 q^{-1}}$$

In the frequency domain $G(q)$ can be written as:

$$G(\omega) = \frac{1 - 0.5 e^{-i\omega T}}{1 - 0.4 e^{-i\omega T}}$$

where T is the sampling time of the process.
The spectrum $\Phi_w(\omega)$ of the signal $w(t)$ is given by:

$\Phi_w(\omega) = |G(\omega)|^2 \, \Phi_e(\omega)$, where $\Phi_e(\omega)$ is the spectrum of $e(t)$.

$$\left| G(\omega) \right|^2 = G(\omega) \cdot \overline{G(\omega)} \quad \text{with} \quad \overline{G(\omega)} = \frac{1 - 0.5\, e^{i\omega T}}{1 - 0.4\, e^{i\omega T}}$$

This gives:

$$I_w(\omega) = \frac{1.25 - \cos \omega T}{1.16 - 0.8 \cos \omega T} \cdot \lambda_e$$

Exercise 2

In our case the prediction model is given by
$\hat{y}(t) = \vartheta^T \varphi(t)$ with $\vartheta = [a, b]$ and $\varphi(t) = \begin{bmatrix} -y(t-1) \\ u(t-1) \end{bmatrix}$,

while the data are generated through the model:

$$y(t) = 0.6\, u(t-1) + 0.3\, u(t-2) + v(t)$$

By using a least squares approach, the estimate of the parameter vector is given by

$$\hat{\vartheta}_N = \left(\frac{1}{N} \sum_{t=1}^{N} \varphi(t) \varphi^T(t) \right)^{-1} \frac{1}{N} \sum_{t=1}^{N} \varphi(t) y(t) =$$

$$= \begin{pmatrix} \frac{1}{N} \sum_{t=1}^{N} y^2(t-1) & \frac{1}{N} \sum_{t=1}^{N} -y(t-1)u(t-1) \\ \frac{1}{N} \sum_{t=1}^{N} -y(t-1)u(t-1) & \frac{1}{N} \sum_{t=1}^{N} u^2(t-1) \end{pmatrix}^{-1} \begin{pmatrix} \frac{1}{N} \sum_{t=1}^{N} -y(t-1)y(t) \\ \frac{1}{N} \sum_{t=1}^{N} u(t-1)y(t) \end{pmatrix}$$

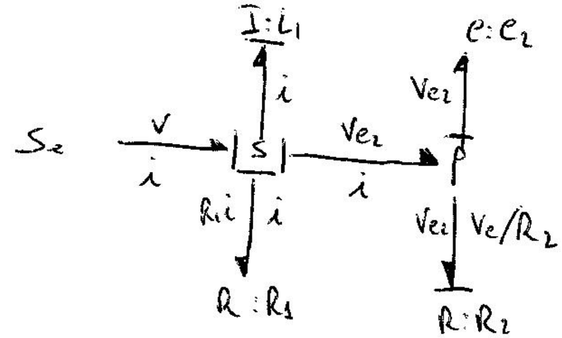
Since $u(t)$ and $y(t)$ are stationary signals, as $N \to \infty$
$\hat{\vartheta}_N \to \hat{\vartheta}$ with:

$$\hat{\vartheta} = \begin{pmatrix} \bar{E}[y^2(t)] & -\bar{E}[y(t)u(t)] \\ -\bar{E}[y(t)u(t)] & \bar{E}[u(t)] \end{pmatrix}^{-1} \begin{pmatrix} -\bar{E}[y(t)y(t-1)] \\ \bar{E}[y(t)u(t-1)] \end{pmatrix} =$$

$$= \begin{pmatrix} 2.45 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -0.18 \\ 0.6 \end{pmatrix} = \begin{bmatrix} -0.07 \\ 0.6 \end{bmatrix}$$

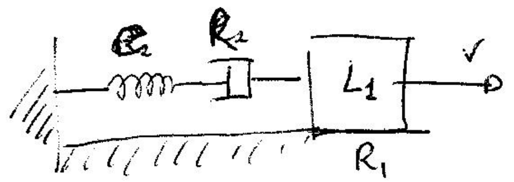
Exercise 3

a) The bond graph for the considered circuit is:



V(t) should be selected as input signal, otherwise a step change in the current signal would result in large voltage in the I element.

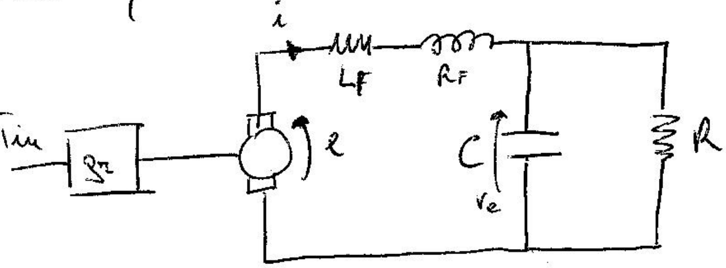
b) A mechanical system with the same bond graph as the previous circuit is:



Note: the same parameters and variables as in the circuit have been chosen

Exercise 4

The system is sketched below.



The corresponding mathematical model in the state space form is:

$$\begin{cases} \dot{V}_e = -\dfrac{V_e}{C} + \dfrac{i}{C} \\ \dot{i} = -\dfrac{V_e}{L_F} - \dfrac{R_F}{L_F} i + \dfrac{K_v}{L_F}\omega \\ \dot{\omega} = -\dfrac{b}{J}\omega + T_m \cdot \dfrac{g_r}{J} \end{cases}$$

Exercise 5

The stability region of the forward Euler method is a circle centered in $(-1/T_s, 0)$ with radius $1/T_s$. I.e., the integration is stable if the eigenvalues of the system lie within this circle. This is not the case for the considered system, integration method and step size. In order to improve the result one can

1) Increase the stability region by decreasing the step size below 0.1 s
2) use the backward Euler method