Solutions to ESS101

080114

1 a) Compact representation of a system.
Cheaper.
Environmentally friendlier.
No other option, system does not exist yet.

b) Determine equilibrium point

$\dot{x} \equiv 0$

$0 = x_2 + u \quad \Rightarrow \quad u = 0 \quad \Rightarrow \quad x_2 = 0$

$0 = (1 - x_1^2)x_2 - x_1 \quad x = 0 \quad \Rightarrow \quad x_1 = 0$

Linearize

$$A = \frac{\partial f}{\partial x}\bigg|_{x_0, u_0} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B = \frac{\partial f}{\partial u}\bigg|_{x_0, u_0} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta \dot{x} = A \Delta x + B \Delta u$$

c) $$RC \frac{dV_{out}}{dt} = - V_{out} + V_{in}$$

Laplace : $V_{out}(s) = \dfrac{1}{1 + sRC} V_{in}(s)$

Identify against a first order system $G(s) = \dfrac{K}{1 + sT}$

Time constant $T = RC = 0{,}003$ (from plot)

$$C = \frac{0{,}003}{R} = \frac{0{,}003}{1000} = \underline{\underline{3 \mu F}}$$

2) $m\ddot{x} = f_1 \cos\theta - f_2 \sin\theta$

$m\ddot{y} = f_1 \sin\theta + f_2 \cos\theta - mg$

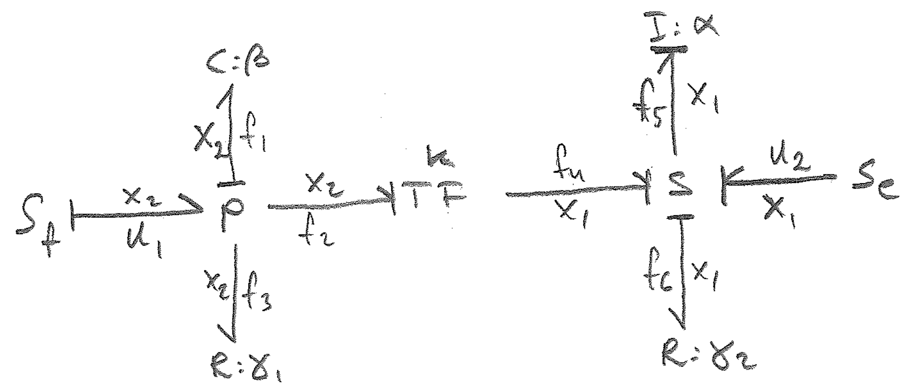
$J\ddot{\theta} = r \cdot f_1$

State variables: $[x, \dot{x}, y, \dot{y}, \theta, \dot{\theta}] = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]$

Input signals f_1, f_2

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{m}(f_1 \cos x_5 - f_2 \sin x_5)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{1}{m}(f_1 \sin x_5 + f_2 \cos x_5) - g$$
$$\dot{x}_5 = x_6$$
$$\dot{x}_6 = \frac{1}{J} r f_1$$

3)



Mark causality, <u>source should be flow source!</u>

Introduce variables in the bond graph

Select state variables : x_1 and x_2

$$\dot{x}_1 = \frac{1}{\alpha} f_5 = \frac{1}{\alpha}(f_4 + u_2 - f_6) = \frac{1}{\alpha}(k x_2 + u_2 - \gamma_2 x_1)$$ ok!

$$\dot{x}_2 = \frac{1}{\beta} f_1 = \frac{1}{\beta}(u_1 - f_2 - f_3) = \frac{1}{\beta}\left(u_1 - k x_1 - \frac{x_2}{\gamma_1}\right)$$ ok!

4) a) If the model is written on the form $y(t) = \theta^T \varphi(t)$ where $\theta = [b_1 \;\; b_2]^T$ and $\varphi(t) = [u(t-1) \;\; u(t-2)]^T$ is the variance for the paramterestimations given as

$$E\{(\theta_N - \theta_0)(\theta_N - \theta_0)^T\} = \frac{\lambda}{N} \bar{R}^{-1}$$

$$\bar{R} = \lim_{N\to\infty} R_N = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \varphi_N(t) \varphi_N^T(t) =$$

$$= \lim_{N\to\infty} \frac{1}{N} \begin{bmatrix} \sum_1^N u^2(t-1) & \sum_1^N u(t-1)u(t-2) \\ \sum_1^N u(t-2)u(t-2) & \sum_1^N u^2(t-2) \end{bmatrix} = \begin{bmatrix} R_u(0) & R_u(1) \\ R_u(1) & R_u(0) \end{bmatrix}$$

$$\bar{R}^{-1} = \frac{1}{R_u^2(0) - R_u^2(1)} \begin{bmatrix} R_u(0) & -R_u(1) \\ -R_u(1) & R_u(0) \end{bmatrix}$$

This shows ~~that~~ the variance only depend on $R_u(0)$ and $R_u(1)$

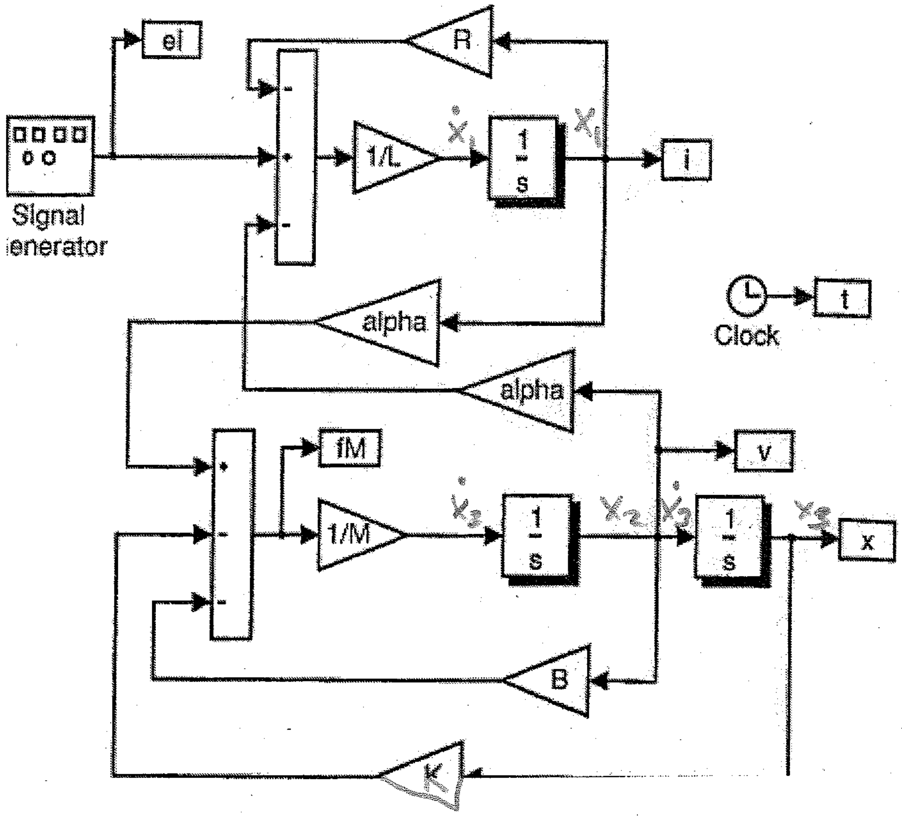
The varianc for $\hat{b}_1$ is $\frac{\lambda}{N} \frac{R_u(0)}{R_u^2(0) - R_u^2(1)}$

b) if $R_u(0) = 1$, then the variance for $\hat{b}_1$ is $\frac{\lambda}{N} \frac{1}{1 - R_u^2(1)}$ which has its smalest value if $R_u(1) = 0$.

Choose $u(t)$ as white noise.

5

a) Introduce state variables, for example as shown in the figure below



From Simulink scheme:

$$\dot{x}_1 = \frac{1}{L}(-Rx_1 + u - \alpha x_2)$$

$$\dot{x}_2 = \frac{1}{M}(\alpha x_1 - K x_3 - B x_2)$$

$$\dot{x}_3 = x_2$$

5 b) Euler method:

$$x_{n+1} = x_n + hf(x) = x_n + h\lambda_i x_n = (1+h\lambda_i)^n x_0$$

where λ_i corresponds to the "fastest" eigenvalue in our case $\lambda_i = -98135$

stable simulation if $|1+h\lambda_i| < 1$

$$-1 < (1+h\lambda_i) < 1$$

$-1 < 1 + h\lambda_i$

$-2 < h\lambda_i$

$h < \frac{-2}{\lambda_i} = \frac{2}{98135}$

$1 + h\lambda_i < 1$

$h\lambda_i < 0$

$h > 0$

$$0 < h < \frac{2}{98135}$$