**Chalmers University of Technology**
**Department of Signals and Systems**

# ESS101 Modelling and simulation

Examination date 071022

***Time:*** 08.30 – 12.30

***Teacher:*** Jonas Fredriksson, tel 031-772 1359.

***Allowed material during the exam:*** Mathematics Handbook.

The exam consists of 5 exercises of a total of 25 points. Nominal grading according to 12/17/21 points, you need 12 points to pass the course with grade 3, 17 points to pass with grade 4 and 21 to pass the course with grade 5. Solutions and answers should be written in English and be unambiguous and well motivated, but preferably short and concise.

Results are announced on the notice board at the latest Nov 5. You may check your grading of your exam on Nov 5 at 12.15-13.15 at the Department of Signals and Systems.

GOOD LUCK!

**Exercise 1** (5 p)

**(a)** Why is it more difficult to estimate an OE-model compared with an ARX-model? (1p)

**(b)** What is the index of the following DAE? (2p)

$$\begin{aligned} \dot{x}_1 + 4x_1 &= u \\ \dot{x}_3 + x_2 &= 2u \\ x_3 &= 3u \\ x_4 &= u \end{aligned}$$

**(c)** Calculate the spectrum $\Phi_y(\omega)$ for

$$y(t) = G(p)u(t) + e(t) \qquad\qquad G(p) = \frac{1}{p+1}$$

when $u(t)$ and $e(t)$ are white noises with variance 1. $u(t)$ and $e(t)$ can be assumed to be uncorrelated. (2p)

**Exercise 2** (5 p)

Consider the model

$$y(t) = a + bt + e(t)$$

where {e(t)} is a sequence of uncorrelated normal distributed with zero mean and variance 1.

**(a)** Determine the least squares estimate of $a$ and $b$ based on the measurements

$$\sum_{t=1}^{4} y(t) = 5 \qquad\qquad \sum_{t=1}^{4} ty(t) = 10$$

(3p)

**(b)** Show that the variance for the estimate of the parameter $a$ decay as $N^{-1}$ while the variance of the estimate of $b$ decay as $N^{-3}$ (2p)

*Hints!* The least squares estimate is given as

$$\hat{\theta} = \left( \frac{1}{N} \sum_{t=1}^{N} \varphi(t)\varphi^T(t) \right)^{-1} \left( \frac{1}{N} \sum_{t=1}^{N} \varphi(t) y(t) \right)$$

and the parameter estimate variance is given as

$$E\{(\theta_N - \theta_0)(\theta_N - \theta_0)^T\} = \frac{\lambda}{N} \bar{R}^{-1}$$

where

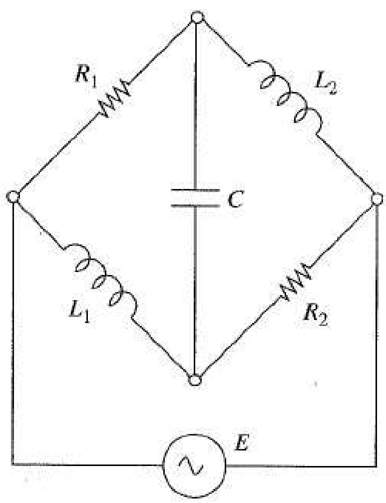
$$\bar{R} = E\{\psi(t)\psi^T(t)\} \qquad \psi(t) = \frac{d}{d\theta} \hat{y}(t|\theta)$$

It could also be good to know that

$$\sum_{t=1}^{N} t = \frac{N(N+1)}{2} \qquad \sum_{t=1}^{N} t^2 = \frac{N(N+1)(2N+1)}{6}$$

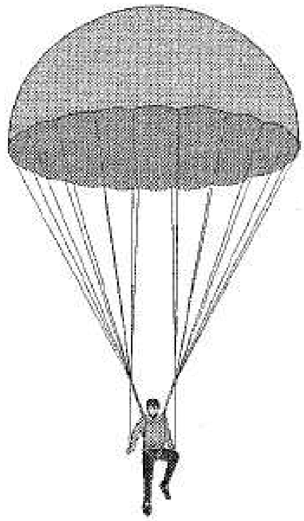
**Exercise 3** (5 p)

Draw a bond graph for the circuit below. Mark causality! Is it possible to describe the system as a system of ordinary first order differential equations?



**Exercise 4** (5 p)

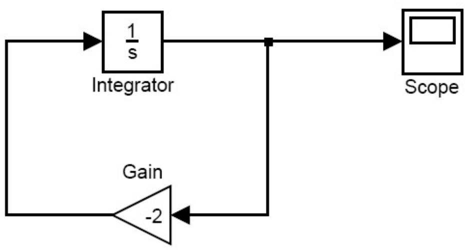
A person wearing a parachute jumps out of a airplane. Assume that there is no wind and that the parachute provides viscous damping relative to a fixed reference frame. The parachute has a small mass $m_p$ and a large damping coefficient $b_p$. The jumper has a larger mass $m_j$ and a smaller drag coefficient $b_j$. The cords attaching the parachute to the jumper are called risers and are assumed to be quite springy. The elastic effect of the risers is represented by the spring constant $k_r$. The deformation of the parachute itself can also be included in the value of $k_r$.



Determine a state space model on the form $\dot{x} = f(x, u)$ for the parachutist!

**Exercise 5** (5 p)

Consider the Simulink model below:



**(a)** You would like to simulate the system using a Runge-Kutta method

$$\begin{aligned} k_1(t) &= f(x(t)) \\ k_2(t) &= f(x(t) + \frac{h}{2}k_1(t)) \\ x(t+h) &= x(t) + hk_2(t) \end{aligned}$$

which is the largest step size that can be used in order to have a stable simulation? (4p)

**(b)** What value on $x(0.2)$ and $x(0.4)$ do you get when using a step size of $h = 0.2$ ($x(0)$=1)? (1p)