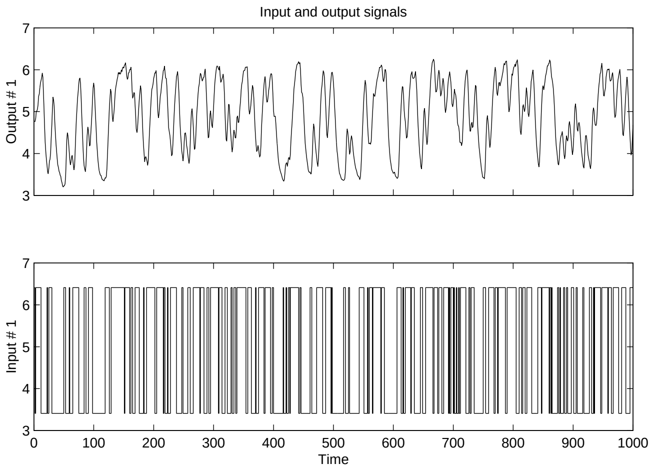
ESS100 Modelling and simulation
Exam Thu, 30 August 2007

Uppgift 1 (10 p)

(a) You have collected data, in order to estimate a linear ready-made model, according to the figure below. What is your next step in the identification process and why? (2p)



(b) What is the difference between a static and dynamic system? (2p)

(c) What is transient analysis? Give example of how it can be used! (2p)

(d) Causality is a central in bond graph modelling. Why is the rule for the causality stroke for an effort storage as it is? (2p)

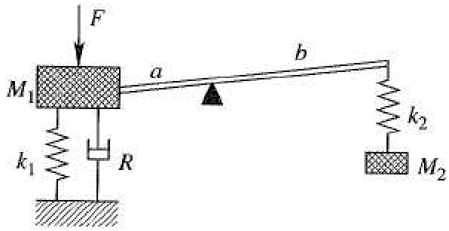
(e) How can the parameters a and b in the system

$$\frac{a}{s+b}$$

be estimated from a step response? (2p)

Uppgift 2 (10 p)

The figure below shows a mechanical system.



Figur 1: Mechanical system

(a) Draw a bond graph for the system? (5p)

(b) Determine a state space model for the system. (5p)

Uppgift 3 (10 p)

Assume that a system that is described by the following difference equation

$$\begin{aligned} &y(t) + a_1 y(t-1) + a_2 y(t-2) + \ldots + a_n y(t-n) \\ &= b_1 u(t-1) + b_2 u(t-2) + \ldots + b_n u(t-n) + K \end{aligned}$$

is to be identified. The constant K is unknown, this can be the case when a sensor gives a constant measurement error (bias). We are interested in determining the parameters a_i and b_i using the least squares method.

(a) Show that regardless of the value of K it is possible to determine the parameters a_i and b_i using least squares by considering the differences

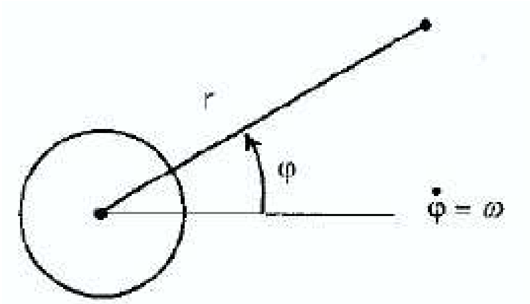
$$\Delta u(t) = u(t) - u(t-1)$$

$$\Delta y(t) = y(t) - y(t-1)$$

as new input- and output signals. (4p)

(b) Show that it is possible to include K in the regular least squares formulation $\hat{y}(t,\theta) = \theta^T \varphi(t)$ so that a_i, b_i and K can be estimated. (6p)

Uppgift 4 (10 p)



Figur 2: Satellite.

For a satellite in a central orbit in a gravitational field the following differential equation holds:

$$\ddot{r} + \frac{k}{r^2} - r\omega = u_1$$

$$\frac{1}{r}\frac{d}{dt}(r^2 w) = u_2$$

where k is the gravitational constant, u_1 and u_2 are radial and tangential control forces.

(a) Determine a state space model for the satellite, with r, $\dot{r}$ and ω as state variables. (5p)

(b) Linearize the satellite around the a nominal orbit with constant angular velocity $\omega_0 = \frac{2\pi}{24*3600}$ rad/s. Perform the linearization. Nominal control forces: $u_{10} = u_{20} = 0$. (5p)

Uppgift 5 (10 p)

It is of extreme importance how one raises a diver from large depths. One has to consider the large pressure differences inside the divers body can cause injuries, decompression sickness, or the diver could even explode. We would like to determine a mathematical model for the diver, when the diver is lifted by an outer force, $F_{lift}(t)$, from a given depth.
Let $h(t)$ denote the diver's depth, m diver's mass, v diver's volume. Assume for simplicity the the density of the water ρ is constant.
The following relationships holds:

- The lifting force from the water is $g(\rho v - m)$.
- The friction force in water is proportional to the divers velocity.

An important physiological variable is the average internal pressure in the diver's tissues. Let $p(t)$ denote this pressure relative to the atmospheric pressure at sea level. The underwater pressure at depth h is ρh relative to atmospheric pressure. Then the following equation for the pressure $p(t)$ holds:

$$\frac{dp(t)}{dt} = k(\rho g h(t) - p(t))$$

where k is a physiologic constant. Another important variable is the pressure difference between the bodies tissues and the surrounding pressure, i.e

$$q(t) = p(t) - \rho g h(t)$$

(a) Determine a mathematical model for the above stated problem, with $F_{lift}(t)$ as input signal and $q(t)$ as output signal. (7p)

(b) State a demand on the lifting force $F_{lift}(t)$ so that there exists a stationary pointy. Determine this stationary point. (3p)