ESS100 Modelling and simulation
Solutions to exam Thu, 30 August 2007

Exercise 1

(a) Remove mean values from the signals. The system identification methods works on systems with zero mean.

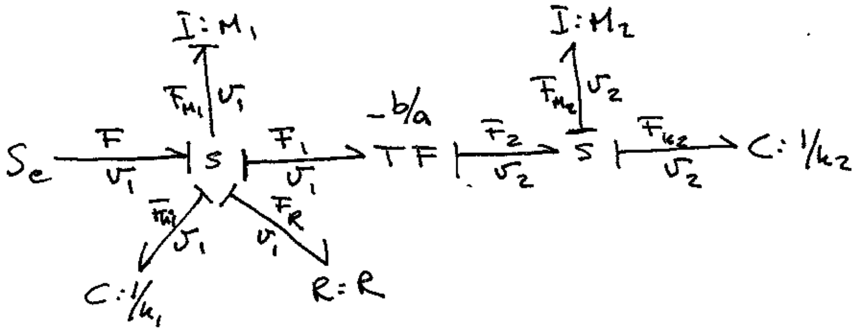
(b) In a static system the output depends directly on the input (no memory), while in a dynamic system the output depend on old input signals and output signals.

(c) A nonparametric identification method, where you study impulse responses of step responses. Example: From a step response you can determine time delay, static gain and time constant.

(d) Since integration is a more natural operation than differentiation, it is natural to have the causality as it is. What happens when the input is a step? Differentiation of a step will lead to an infinite large output signal, while integration will lead to a nice output signal.

(e) From the step response can the static gain and the time constant be determined. Static gain is given as a/b and the time constant as $1/b$.

2. a) Bond graph.



Conflict in causality rules

b) State space model

State variables : v_1 , F_{k_1}, v_2, F_{k_2}, F_{M_2} extra state due to conflict in causality

$$\dot{v}_1 = \frac{1}{M_1} F_{M_1} = \frac{1}{M_1}\left(F - F_1 - F_R - F_{k_1}\right) = \frac{1}{M_1}\left(F + \frac{a}{b} F_2 - R v_1 - F_{k_1}\right)$$

$$= \frac{1}{M_1}\left(F + \frac{a}{b}\left(F_{M_2} + F_{k_2}\right) - R v_1 - F_{k_1}\right) =$$

$$\dot{F}_{k_1} = k_1 v_1$$

$$\dot{v}_2 = \frac{1}{M_2} F_{M_2}$$

$$\dot{F}_{k_2} = k_2 v_2$$

3 a)

$$y(t) = -a_1\, y(t-1) - a_2 y(t-2) - \dots - a_n y(t-n)$$
$$+ b_1 u(t-1) + b_2 u(t-2) + \dots + b_n u(t-n) + K$$

$$y(t-1) = -a_1 y(t-2) - a_2 y(t-3) - \dots - a_n y(t-n-1) +$$
$$+ b_1 u(t-2) + b_2 u(t-2) + \dots + b_n u(t-n-1) + K$$

$$\Delta y(t) = y(t) - y(t-1) = -a_1\, y(t-1) - a_2 y(t-2) - \dots - a_n y(t-n)$$
$$+ b_1\, u(t-1) + b_2 u(t-2) + \dots + b_n u(t-n) + K$$
$$+ a_1 y(t-2) + a_2 y(t-3) + \dots + a_n\, y(t-n-1) -$$
$$- b_1\, u(t-2) - b_2\, u(t-3) + \dots - b_n u(t-n-1) - K$$
$$= -a_1 \Delta y(t-1) - a_2 \Delta y(t-2) - \dots - a_n \Delta y(t-n)$$
$$+ b_1\, \Delta u(t-1) + b_2 \Delta u(t-2) + \dots + b_n \Delta u(t-n)$$

i.e. a linear difference equation with the same parameters as the original system, but K has been eliminated.

b) Let

$$\theta^T = [\, a_1 \dots a_n \; b_1 \dots b_n \; K \,]$$
$$\varphi(t) = [-y(t-1) \; \dots -y(t-n) \; u(t-1) \; \dots u(t-n) \;\; 1]^T$$

Exercise 4

(a) Let $r = x_1$, $\dot{r} = x_2$ and $\omega = x_3$

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_1x_3^2 - \frac{k}{x_1^2} + u_1\\ \dot{x}_3 &= -\frac{2x_2x_3}{x_1} + \frac{u_2}{x_1}\end{aligned}$$

(b) Stationary point $x_{30} = \omega_0$, $x_{20} = 0$ and $x_{10} = (k/\omega_0^2)^(1/3)$

$$\Delta\dot{x} = \begin{bmatrix} 0 & 1 & 0\\ x_{30}^2 + \frac{2k}{x_{10}^3} & 0 & 2x_{10}x_{30}\\ 0 & -\frac{2x_{30}}{x_{10}} & 0\end{bmatrix}\Delta x + \begin{bmatrix} 0 & 0\\ 1 & 0\\ 0 & \frac{1}{x_{10}}\end{bmatrix}\Delta u$$

Exercise 5

(a) With $x_1 = h, x_2 = \dot{h}, x_3 = p, u = F_{lyft}$ and $y = q$, the following state-space model can be formulated:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= -\tfrac{b}{m}x_2(t) - \tfrac{1}{m}u(t) - g(\tfrac{\rho v}{m} - 1)\\ \dot{x}_3(t) &= k\rho g x_1(t) - kx_3(t)\\ y(t) &= -\rho g x_1(t) + x_3(t).\end{aligned}$$

(b) Stationary point: $x = \begin{bmatrix} h_0 & 0 & \rho g h_0\end{bmatrix}^T$. Stationarity implies $u = g(m - \rho v)$.