ESS100 Modelling and simulation
Exam Thu, 12 April 2007

Exercise 1 (10 p)

(a) What is transient analysis? Give example of how it can be used! (2p)

(b) In regular simulation softwares it is only possible to simulated ordinary differential equations. Mention one way to make it possible to simulate partial differential equations in these softwares. (2p)

(c) When identifying a black-box model you can choose a model of high order, i.e. many parameters. Mention one good thing and one bad thing with this choice! (2p)

(d) What is the index of the following DAE?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

(2p)

(e) What is the difference between a static and a dynamic system? (2p)

Exercise 2 (10 p)

Assume that a model with the following model structure

$$y(t) = bu(t-1) + e(t)$$

is adapted to measurement data. The true data is generated from the system

$$y(t) = b_1 u(t-1) + b_2 u(t-2) + e(t)$$

, where $e(t)$ is white noise with variance λ. To which value does the least squares estimation of b (expressed in terms of b_1 and b_2) converge, when the number of observations goes to infinity and

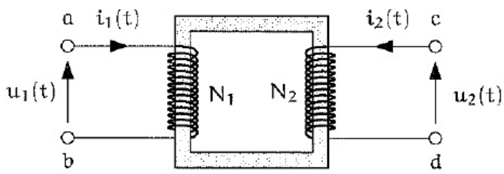
(a) $u(t)$ is white noise with variance 1. (5p)

(b) $u(t)$ has the covariance function (5p)

$$\begin{aligned} R_u(0) &= 1 \\ R_u(1) &= 0.5 \\ R_u(2) &= 0.25 \\ R_u(3) &= 0.125 \\ &\vdots \end{aligned}$$

Exercise 3 (10 p)

Below is a sketch of a transformer with an iron core.



The transformer can be modelled in the following way:

- The voltages are proportional to the magnetic flux:

$$u_1(t) = N_1\dot{\Phi}(t) \quad u_2(t) = N_2\dot{\Phi}(t)$$

  where N_1 and N_2 are the number of turns.

- The magnetic field $B(t)$ per area unit is

$$B(t) = \frac{\Phi(t)}{A}$$

  where A is assumed to be constant.

- If $i_m(t)$ denote the so-called magnetizing current, then

$$N_1 i_1(t) - N_2 i_2(t) = N_1 i_m(t)$$

- The magnetizing current is a nonlinear function of the magnetic field:

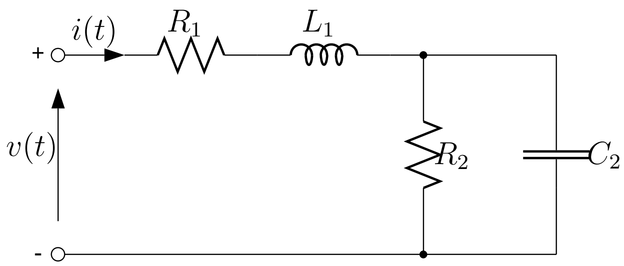
$$i_m(t) = \varphi(B(t))$$

(a) Assume that a resistor R is attached between the points c and d on the transformers secondary side. Determine a state space model on the form $\dot{x} = f(x, u), y = h(x, u)$ for the system, with i_1 as input signal and i_2 as output signal. (6p)

(b) $\varphi(B(t))$ can be assumed to be linear in a small region. Linearize the system in a point in this region. (4p)

Exercise 4 (10 p)

Consider the electric circuit below.



(a) Draw a bond graph and mark causality for the circuit. Should $v(t)$ or $i(t)$ be chosen as input signal? Motivate! (5p)

(b) Describe a mechanical system which would have a similar bond graph as the electric circuit. (5p)

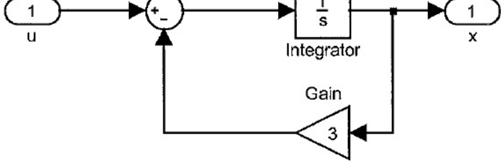
Exercise 5 (10 p)

A hydraulic servo can be described as

$$\frac{1}{4}y^{(3)}(t) + \frac{1}{2}\ddot{y}(t) + \dot{y}(t) = u(t)$$

(a) Draw a Simulink block diagram which represent the servo Let $u(t)$ be input signal and $y(t)$ be output signal. (5p)

(b) Consider the Simulink model below::



You would like to simulate the system using a backward Euler method ($x(t+h) = x(t) + hf(x(t+h))$), which is the longest step size that can be used in order to have a stable simulation? The input signal u can be assumed to be zero. (5p)