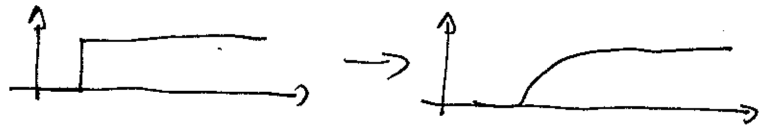
Solutions to the exam ESS101

2007-04-12

1. a) A nonparametric method for identification, where you study impulse responses or step responses

Example:



b) Discretize the spacial variable

c) High accuracy, but sensitive to disturbances which are modelled (or captured) in the model.

d) On Kronecker form

$$N = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$ index $= k$ $\quad N^k = 0$ gives

$N^3 = 0 \Rightarrow k = 3$ $\underline{\underline{\text{index} = 3}}$

e) In a static system the output depends directly on the input (no memory), while in a dynamic system the output depends on old input signals and output signals.

2.

a) $E\{y^2(t)\} = E\{(b_1 u(t-1) + b_2 u(t-2) + e(t))^2\} =$

$= b_1^2 + b_2^2 + \lambda$

$E\{y(t) u(t-1)\} = b_1$

$E\{(y - y_m)^2\} = b_1^2 + b_2^2 + \lambda - 2 b b_1 + b^2 + \lambda$

$= (b - b_1)^2 + b_2^2 + 2\lambda$

min on $\underline{\underline{b = b_1}}$

b) $E\{y^2(t)\} = b_1^2 E\{u^2(t-1)\} + 2 b_1 b_2 E\{u(t-1) u(t-2)\}$

$+ b_2^2 E\{u^2(t-2)\} + \lambda = b_1^2 + b_1 b_2 + b_2^2 + \lambda$

$E\{y(t) u(t-1)\} = b_1 + 0{,}5 b_2$

$E\{(y - y_m)^2\} = b_1^2 + b_1 b_2 + b_2^2 + \lambda - 2 b b_1 - b b_2 + b^2 + \lambda$

$= b_1^2 + b_1 b_2 + b_2^2 + 2\lambda - 2 b b_1 - b b_2 + b^2$

$-2 b_1 - b_2 + 2 b = 0$

$$\underline{\underline{b = \frac{2 b_1 + b_2}{2}}}$$

$$u_2 = R\, i_2$$

3\. a)

$$i_2 = \frac{N_1}{N_2} i_1 - \frac{N_1}{N_2} i_m = \frac{N_1}{N_2} i_1 - \frac{N_1}{N_2} \varphi_m(B)$$

$$\dot{B} = \frac{\dot{\Phi}}{A} = \frac{1}{N_2 A} u_2 = \frac{R}{N_2 A} i_2 = \underline{\underline{\frac{R}{N_2 A}\left(\frac{N_1}{N_2} i_1 - \frac{N_1}{N_2} \varphi_m(B)\right)}}$$

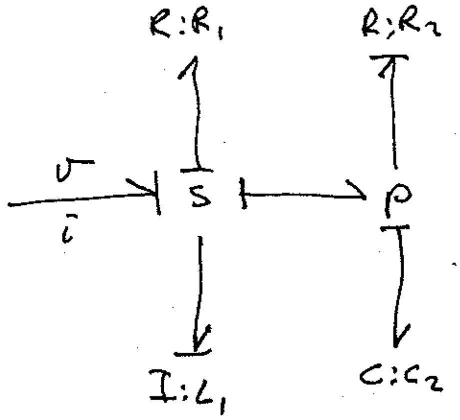
$$\underline{\underline{i_2 = \frac{N_1}{N_2} i_1 - \frac{N_1}{N_2} \varphi_m(B)}}$$

b) Linearize

$$\Delta\dot{B} = - \frac{R N_1}{A N_2^2} k\, \Delta B + \frac{R N_1}{A N_2^2} \Delta i_1$$

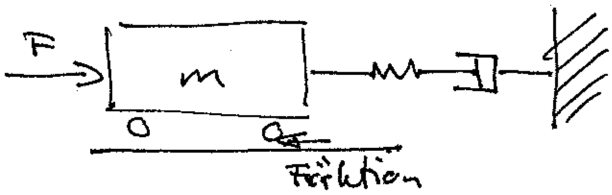
$$\Delta i_2 = - \frac{N_1}{N_2} k\, \Delta B + \frac{N_1}{N_2} \Delta i_1$$

4 a)



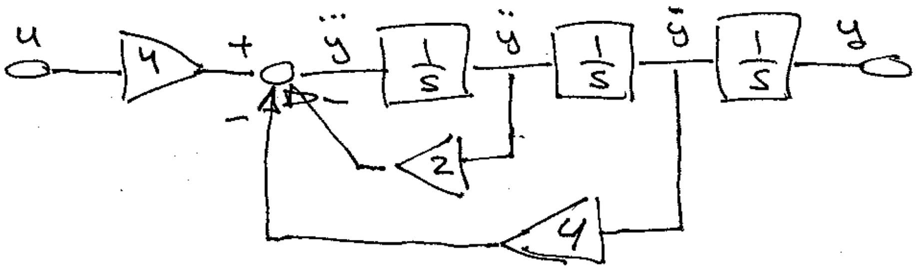
v input signal → causality ok.

b)



5.

a



b)

$$\dot{x} = -3x$$

$$x(t+h) = x(t) - 3h\,x(t+h)$$

$$x(t+h) = \frac{1}{1-3h}\,x(t)$$

Stable if:

$$\left|\frac{1}{1-3h}\right| < 1$$

$$|1-3h| > 1$$

$$1-3h < -1$$

$$-3h < -2$$

$$\underline{\underline{h > \frac{2}{3}}}$$