ESS100 Modelling and simulation
Exam Thu, 31 March 2005

Exercise 1 (10 p)

(a) What is the difference between a dynamic and a static system? (2p)

(b) When identifying a black-box model you can choose a model of high order, i.e. many parameters. Mention one good thing and one bad thing with this choice! (2p)

(c) In regular simulation softwares it is only possible to simulated ordinary differential equations. Mention one way to make it possible to simulate partial differential equations in these softwares. (2p)

(d) What is the index of the following DAE system?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

(2p)

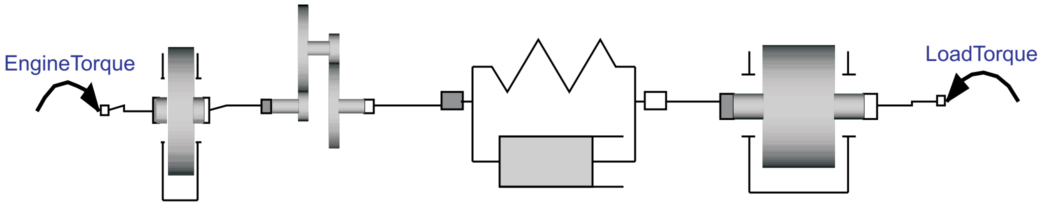
(e) How can the parameters a and b in the system

$$\frac{b}{s+a}$$

be determined from a step response? (2p)

Exercise 2 (7 p)

A vehicle driveline can be described according to the figure below
The two inertias correspond to the engine's flywheel and the wheels. The gear ratio corresponds to the transmission, the spring and damper system describes the driveline's axles.



Figur 1: Driveline model in Modelica.

(a) Determine a state space representation for the driveline, on the form $\dot{x} = Ax + Bu,\ y = Cx + Du$. Let the engine torque be the input signal and the wheel's (vehicle's) longitudinal velocity be the output. Introduce state variables and suitable notations for the all parameters in the driveline. (5p)

(b) In vehicle dynamics and tire modelling *slip* is a commonly used concept. Slip is defined as

$$slip = \frac{v_{fordon} - v_{hjul}}{v_{fordon}}$$

i.e. the relative velocity difference between wheel and vehicle. Which problem can arise when using this definition is simulations? (2p)

Exercise 3 (10 p)

A system is described as:

$$\frac{d^2y}{dt^2} + y\frac{dy}{dt} - 3u = 1$$

(a) Introduce suitable state variables and write the system as a system of first order differential equations. (3p)

(b) Linearize the system around the stationary point $y_0 = 1$. (4p)

(c) You would like to simulate the system in Simulink. Draw an equivalent block diagram (Simulink scheme) for the system. (3p)

Exercise 4 (10 p)

A system which you would like to identify is described as

$$y(t) = 0.6u(t-1) + 0.4u(t-2) + v(t)$$

where v is white noise with variance λ. You adapt the model

$$y(t) = bu(t-k) + e(t)$$

to measurement data using the prediction error method. The input signal u has covariance function

$$R_u(0) = 1, \quad R_u(1) = 0.5, \quad R_u(\tau) = 0, \ |\tau| > 1$$

To which value will the estimate of b converge, when the number of measurements goes to infinity? Examine the cases $k = 1, k = 2, k = 3$ and $k > 3$.

Exercise 5 (5 p)

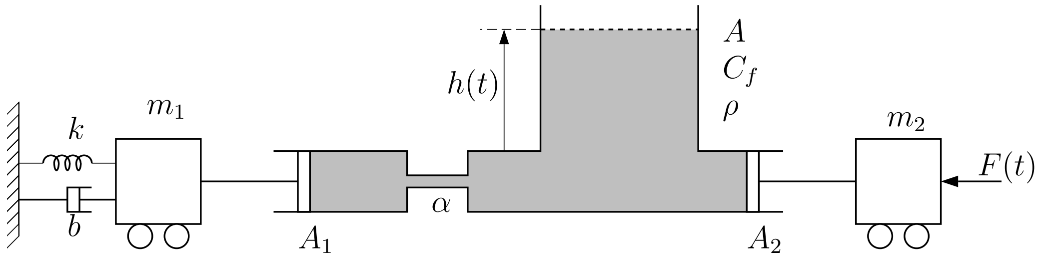
Let $\{w(t)\}$ be a stationary stochastic process. $w(t)$ is generated from the time discrete relation

$$w(t) - 0.6w(t-1) = e(t) - 0.4e(t-1)$$

where $\{e(t)\}$ is white noise with variance λ_e. Determine the spectrum $\Phi(\omega)$ for $\{w(t)\}$

Exercise 6 (8 p)

Regard the hydraulic-mechanical system shown below. The purpose of the system is control the level in the tank by using the force $F(t)$.



$$
\begin{aligned}
F(t) &= \text{ force asserted on the mass } m_2 [N] && (1)\\
m_1, m_2 &= \text{ masses } [kg] && (2)\\
k &= \text{ spring constant } [N/m] && (3)\\
b &= \text{ damping coefficient } [Nm] && (4)\\
A, A_1, A_2 &= \text{ areas } [m^2] && (5)\\
\rho &= \text{ density of the fluid } [kg/m^3] && (6)\\
C_f &= \text{ fluid capacitance } [m^4 s^2/kg] && (7)
\end{aligned}
$$

The energy loss in the pipes due to friction can be described as

$$p_s(t) = \alpha Q_s(t)$$

where

$$
\begin{aligned}
p_s(t) &= \text{ pressure difference } [N/m^2]\\
Q_s(t) &= \text{ flow through the pipe } [m^3/s]
\end{aligned}
$$

(a) Draw a bond graph for the system and determine causality.

(b) Determine input and output signals and indicate a suitable choice of state variables if the system is to describe in state space form.