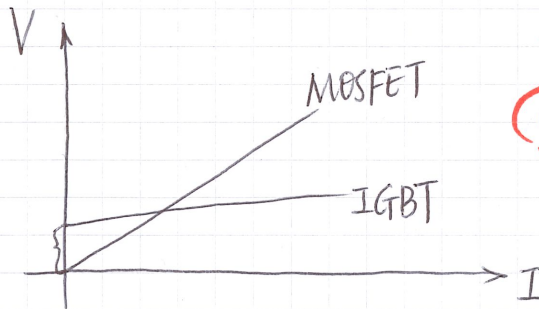


Q # 1.

(a) Fourier components represent sinusoidal components with frequencies multiples of the signal frequency of a periodic signal. ①

We need to calculate them to know the harmonic content (THD) of a signal to design the corresponding filters and avoid electromagnetic interference (EMI). ①

(b) IGBT : smaller  $R_{on}$ , non-zero threshold conduction voltage. ①  
MOSFET : higher  $R_{on}$ , zero threshold conduction voltage.



Q#2.

$$(a) \quad N_1:N_2:N_3 = 1:2:2, \quad V_d = 20V, \quad f_{sw} = 20 \text{ kHz}$$

$$D = 0.3, \quad L_m = 100 \mu\text{H}, \quad R_{load} = 20\Omega, \quad 60\Omega. \quad V_o?$$

First identify CCM or DCM.

Assume CCM.

$$\Rightarrow \frac{V_o}{V_d} = \frac{N_3}{N_1} \frac{D}{1-D}$$

$$\Rightarrow V_o = \frac{2}{1} \cdot \frac{0.3}{0.7} \cdot 20V = 17.14V < \frac{N_3}{N_2} V_d = 20V$$

$$\Rightarrow I_o = \frac{V_o}{R_{load}} = \begin{cases} 0.857 \text{ A} & \text{for } R_{load} = 20\Omega \\ 0.286 \text{ A} & \text{for } R_{load} = 60\Omega \end{cases}$$

$$P_{in} = P_{out} \Rightarrow V_d I_d = V_o I_o$$

$$\Rightarrow I_d = \begin{cases} 0.734 \text{ A} & \text{for } R_{load} = 20\Omega \\ 0.244 \text{ A} & \text{for } R_{load} = 60\Omega \end{cases}$$

$$I_m = I_d + I_o \frac{N_3}{N_1} = \begin{cases} 2.45 \text{ A} & \text{for } R_{load} = 20\Omega \\ 0.82 \text{ A} & \text{for } R_{load} = 60\Omega \end{cases} \quad (1)$$

$$\frac{\Delta i_m}{2} = \frac{D V_d}{2 L_m f_{sw}} = 1.5 \text{ A}$$

$$\begin{cases} \frac{\Delta i_m}{2} < I_m & \text{for } R_{load} = 20\Omega \quad \text{CCM. } (1) \\ \frac{\Delta i_m}{2} > I_m & \text{for } R_{load} = 60\Omega \quad \text{DCM. } (1) \end{cases}$$

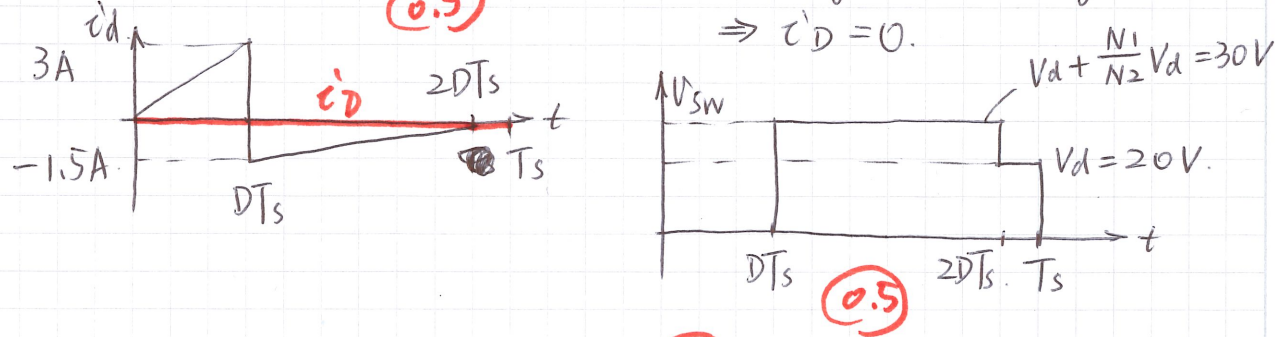
for  $R_{load} = 20\Omega \Rightarrow CCM \Rightarrow V_o = \frac{N_3}{N_1} \cdot \frac{D}{1-D} V_d = 17,14V$   
 $< \frac{N_3}{N_2} V_d = 20V$   
 $\Rightarrow$  winding  $N_2$  not used.

for  $R_{load} = 60\Omega \Rightarrow DCM$   
 $\Rightarrow V_o = D \cdot \sqrt{\frac{R_{load}}{2L_m f_{sw}}} \cdot V_d = 23,24V > \frac{N_3}{N_2} V_d = 20V$   
 $\Rightarrow V_o = V_{o,max} = \frac{N_3}{N_2} V_d = 20V$  ①  
 $\Rightarrow$  winding  $N_2$  is conducting.

(b)  $R_{load} = 20\Omega$ , CCM.  $\Delta i_m = 3A$ ,  $I_m = 1.591A$ .



$R_{load} = 60\Omega$ , DCM.  $\Delta i_m = 3A$ . winding  $N_2$  conducting.



(c) Unipolar core excitation. ①  
 Core with airgap as the transformer is used for energy storage. ①

Q #3.  $N_1 : N_3 : N_2 = 1 : 0.5 : 1$ .  $V_0 = 15V$ ,  $P_0 = 50W$ .  
 $V_d = 25V$ .  $f_{sw} = 20kHz$ .

$$(a). I_0 = \frac{P_0}{V_0} = \frac{50}{15} = 3.33A = I_L$$

$$\Rightarrow \Delta i_L = 0.1 I_0 = 0.333A \Rightarrow \frac{\Delta i_L}{2} < I_L \Rightarrow CCM$$

$$\frac{V_0}{V_d} = \frac{N_2}{N_1} \cdot D \Rightarrow D = 0.6 \quad (1)$$

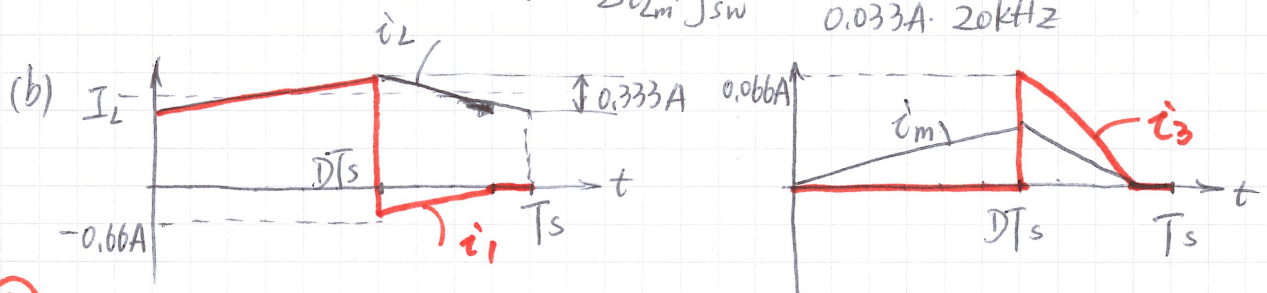
During on period.  $V_L = \frac{N_2}{N_1} V_d - V_0 = V_d - V_0 = \frac{L \cdot \Delta i_L}{DT_s}$  (1)

$$L = \frac{D \cdot (V_d - V_0)}{\Delta i_L \cdot f_{sw}} = \frac{0.6 \cdot (25V - 15V)}{0.333A \cdot 20kHz} = 0.9mH$$

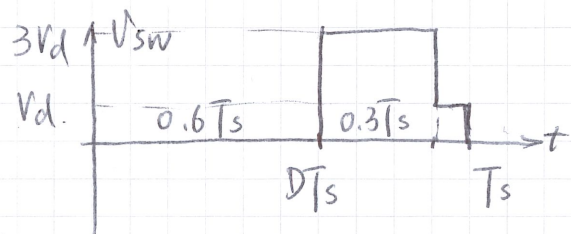
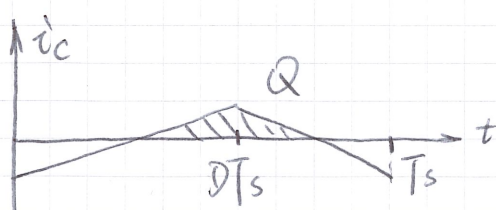
$$\Rightarrow \Delta i_{Lm} = 0.01 I_0 = 0.0333A$$

During on period.  $V_L = V_d = L_m \frac{\Delta i_{Lm}}{DT_s}$

$$L_m = \frac{D \cdot V_d}{\Delta i_{Lm} \cdot f_{sw}} = \frac{0.6 \cdot 25V}{0.0333A \cdot 20kHz} = 22.52mH$$



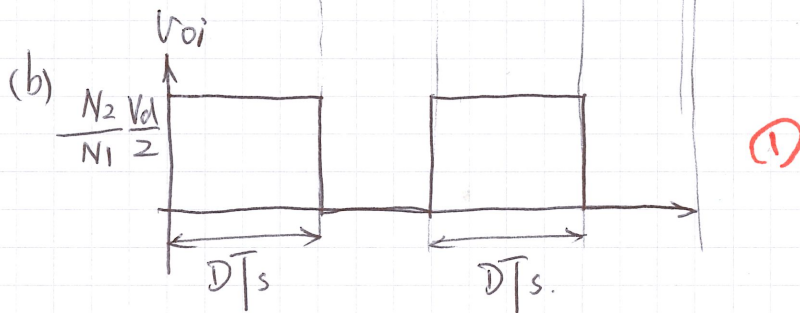
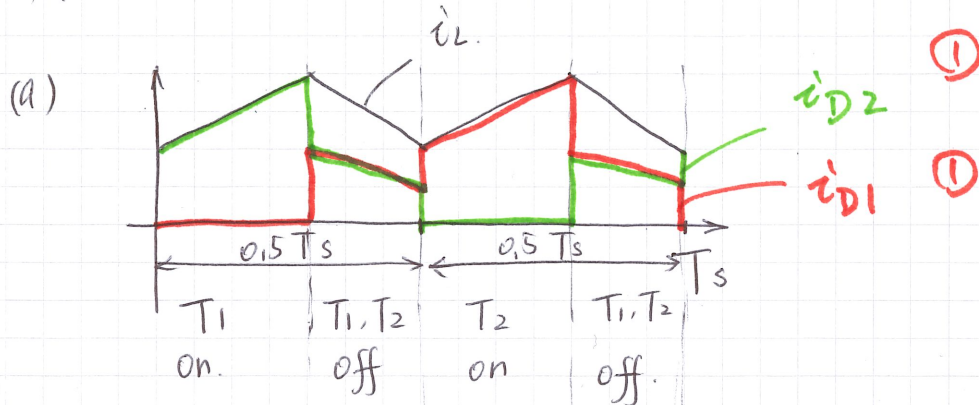
(4)



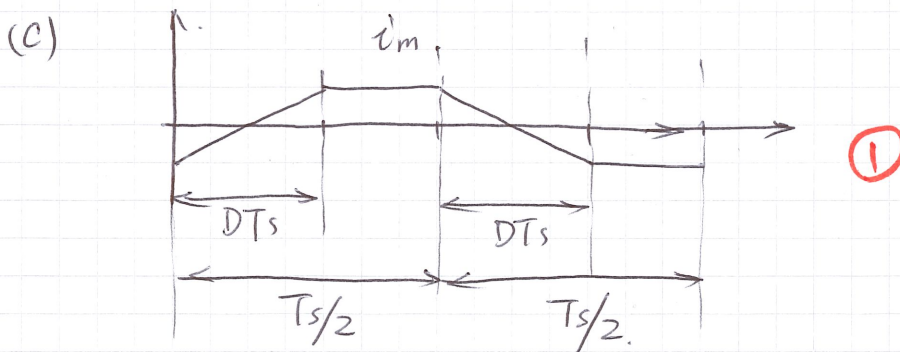
$$(c) \Delta V_0 = 0.01 V_0 = 0.15V = \frac{Q}{C} = \frac{\frac{1}{2} \cdot \frac{\Delta i_L}{2} \cdot \frac{T_s}{2}}{C} \Rightarrow C = 13.9 \mu F \quad (2)$$

(d). Unipolar core excitation. No air gap as  $L_m$  should be large so  $i_m$  is small. The transformer is not used for energy storage as there is another inductance. (2)

Q # 4.

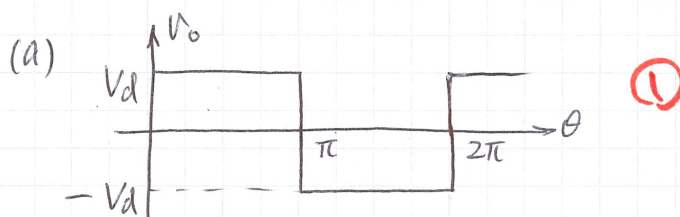


$$V_o = \frac{1}{T_s} \int_0^{T_s} v_{oi} \cdot dt = \frac{2 \cdot DT_s \cdot \frac{N_2}{N_1} \cdot \frac{V_d}{2}}{T_s} = \frac{N_2}{N_1} \cdot V_d \cdot D$$



bipolar core excitation  $\Rightarrow$  higher operating range. ①  
 $\Rightarrow$  core saturation can be avoided.

Q #5



$$\hat{V}_{o,1} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} V_d \cdot \sin\theta \cdot d\theta = V_d \cdot \frac{4}{\pi} \cdot (-\cos\theta) \Big|_0^{\frac{\pi}{2}} = \frac{4}{\pi} V_d.$$

$$V_{o,rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_d^2 \cdot d\theta} = V_d.$$

$$V_{o,1,rms} = \frac{4}{\pi} V_d \cdot \frac{1}{\sqrt{2}} = \frac{2\sqrt{2}}{\pi} V_d.$$

$$\Rightarrow THD = \frac{\sqrt{V_{o,rms}^2 - V_{o,1,rms}^2}}{V_{o,1,rms}} \times 100\%.$$

$$= \frac{\sqrt{1 - \left(\frac{2\sqrt{2}}{\pi}\right)^2}}{\frac{2\sqrt{2}}{\pi}} \times 100\% = 48.3\%.$$

(b) current harmonics

$$\left\{ \begin{array}{l} 1\text{-phase} : 2n \pm 1 = 3, 5, 7, 9, \dots \\ 3\text{-phase} : 6n \pm 1 = 5, 7, 11, 13, \dots \end{array} \right.$$

3-phase removes odd harmonics that are multiples of 3.

(c) Using PWM instead of square-wave - by using a high frequency triangular wave for modulation, the low-order harmonics are pushed higher up.

(d) Multilevel: advantages.  $\Rightarrow$  better harmonics performance.  
 $\Rightarrow$  lower switching losses.

disadvantages.  $\Rightarrow$  more components.  
 $\Rightarrow$  more complicated design.

Q #6.

$$v_{s1} = 230V \frac{\sqrt{2}}{\sqrt{3}} \sin \theta = 230V \frac{\sqrt{2}}{\sqrt{3}} \sin(2\pi \cdot 50\text{Hz} \cdot t)$$

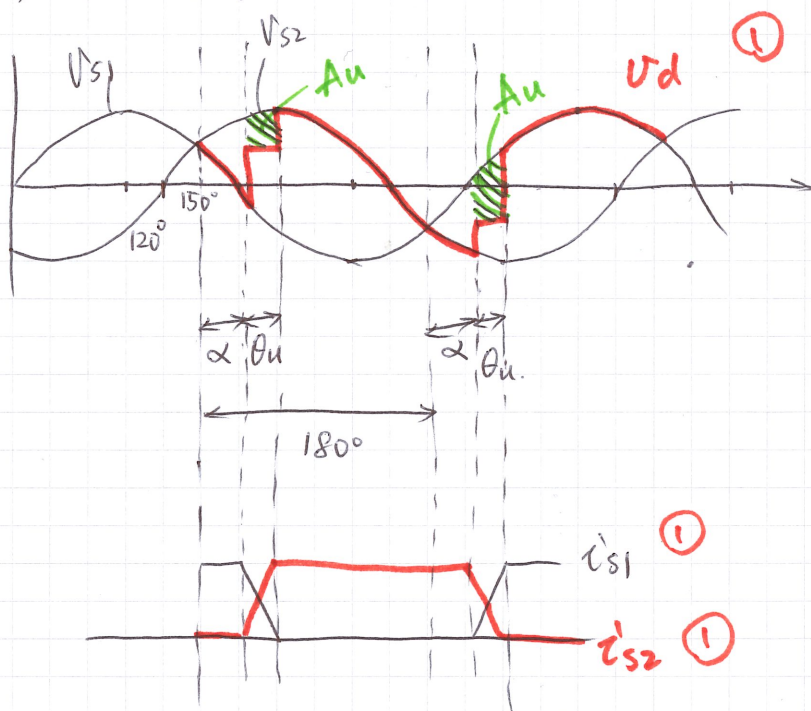
$$v_{s2} = 230V \frac{\sqrt{2}}{\sqrt{3}} \sin(\theta - \frac{2}{3}\pi) = 230V \frac{\sqrt{2}}{\sqrt{3}} \sin(2\pi \cdot 50\text{Hz} \cdot t - \frac{2}{3}\pi)$$

$$L_s = 5\text{mH}$$

$$I_d = 10\text{A}$$

↑  
120°

(a)  $\alpha = 35^\circ$



$$\textcircled{1} V_d = \frac{1}{2\pi} \left( \int_{\alpha+150}^{\alpha+\pi+150} v_{s2} \cdot d\theta + \int_{\alpha+\pi+150}^{\alpha+\pi+150+\pi} v_{s1} \cdot d\theta \right) - \frac{2A_u}{2\pi}$$

$$= \frac{1}{2\pi} \left[ \int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{6}+\pi} v_{s1} \cdot d\theta + \int_{\alpha+\pi+\frac{5\pi}{6}}^{\alpha+\pi+\frac{5\pi}{6}+\pi} v_{s1} \cdot d\theta \right] - \frac{2A_u}{2\pi}$$

$$\textcircled{1} A_u = ? \quad v_{s1} - v_{s2} - L_s \frac{di_{s1}}{dt} + L_s \frac{di_{s2}}{dt} = 0 \quad i_{s1} = I_d - i_u$$

$$\Rightarrow 2L_s \frac{di_u}{dt} = v_{s2} - v_{s1} \quad i_{s2} = i_u$$

$$A_u = \frac{1}{2} \int (v_{s2} - v_{s1}) \cdot d\theta = L_s \omega I_d$$

(b).  $\alpha = 145^\circ \Rightarrow$  average output voltage will be negative. ①  
use previous expression.

$$V_d = \frac{1}{2\pi} \left[ \int_{\alpha + \pi/6}^{\alpha + \pi/6 + \pi} U_{s1} d\theta + \int_{\alpha + 5\pi/6 + \pi}^{\alpha + 5\pi/6 + 2\pi} U_{s1} d\theta \right] - \frac{2L_s \omega_d}{2\pi}$$

$$\alpha = \frac{145}{180} \cdot \pi$$
②

(c). in this case  $\alpha = 0$ .

$$L_s = 0 \Rightarrow A_u = 0$$

use previous expression and no commutation in the plots.

$i_{s1}$ ,  $i_{s2}$  plots. ①

$V_d$  plot. ①

$V_d$  calculation. ①

$$V_d = \frac{1}{2\pi} \left[ \int_{\pi/6}^{\pi/6 + \pi} U_{s1} d\theta + \int_{5\pi/6 + \pi}^{5\pi/6 + 2\pi} U_{s1} d\theta \right]$$