

- (a) diode - uncontrolled: Conducts when forward biased  
 thyristor - semi controlled: turns on when a gate signal is applied and they are forward biased.  
 - turns off with a reverse voltage (2)  
 MOSFET - turns on & off with a gate-source voltage

(b) with an air gap in the inductor, the inductance is given by

$$L = \frac{N^2}{R_c + R_g} \quad \begin{array}{l} R_c = \text{reluctance of core} \\ R_g = \text{reluctance of air gap} \end{array} \quad (1)$$

for a given saturation (maximum) flux density in the core,

$$N i_{\text{sat}} = B_{\text{sat}} A_c (R_g + R_c) \quad (\text{Ampere's law})$$

$$\Rightarrow i_{\text{sat}} = \frac{B_{\text{sat}} A_c}{N} \cdot (R_c + R_g) \quad (2)$$

air gap increases saturation current and hence operating range.

using (1) & (2), it is possible to show that  $W = \frac{1}{2} L i_{\text{sat}}^2$  increases with air gap

- (c) - avoiding circulating currents  
 - safety  
 - multiple outputs  
 - different turns ratios (2)

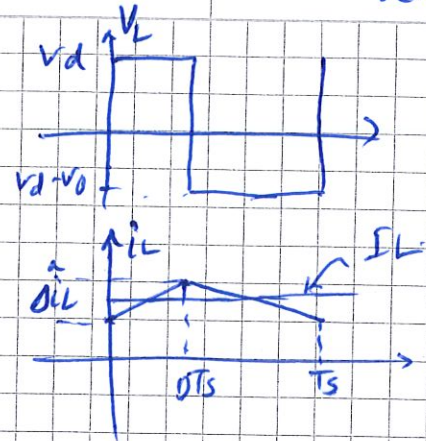
$$V_d = 12V$$

$$P_o = 40W$$

$$V_o = 20V$$

$$f_{sw} = 20kHz$$

Stepup Converter



$$(a) \text{ CCM} \Rightarrow I_L \geq \frac{1}{2} \hat{i}_L$$

$$\text{From } V_d I_L = V_o I_o = P_o$$

$$\text{we have } I_L = \frac{40}{12} A = \underline{\underline{3.33A}}$$

$$\text{CCM} \Rightarrow V_o = \frac{V_d}{1-D} \quad (\text{derive this expression !!})$$

$$\Rightarrow D = 1 - V_d/V_o = \underline{\underline{0.4}}$$

$$\Rightarrow \text{when the switch is on, } V_L = V_d = 12V$$

$$V_L = L \frac{\hat{i}_L}{\Delta t} = L \frac{\hat{i}_L}{D T_s} = L f_{sw} \frac{\hat{i}_L}{D} = V_d \quad (2)$$

$$\frac{\hat{i}_L}{2} \leq I_L \Rightarrow \frac{D V_d}{2 L f_{sw}} \leq I_L = \frac{P_o}{V_d}$$

$$\Rightarrow L \geq \frac{D V_d^2}{2 f_{sw} P_o} = 36 \mu H$$

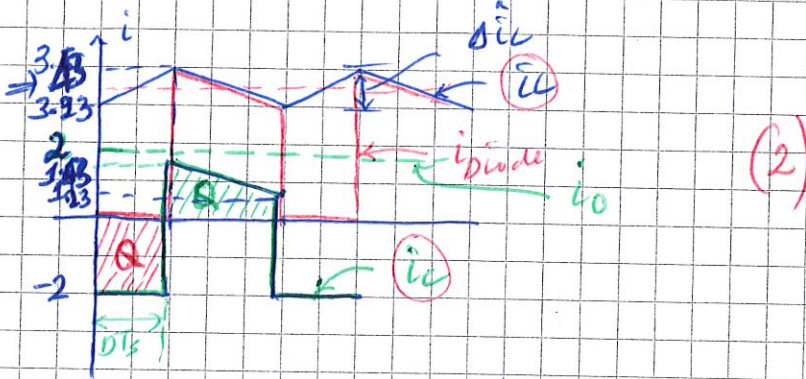
$$\Rightarrow L_{min} = \underline{\underline{36 \mu H}} \quad \text{for CCM}$$

$$(b) \hat{i}_L = 0.1 I_o = 0.1 P_o / V_o \Rightarrow V_L = V_d = L \frac{\hat{i}_L}{D T_s}$$

$$\Rightarrow V_d = \frac{L (0.1 P_o / V_o) f_{sw}}{D} \quad (2)$$

$$\Rightarrow L = \frac{D V_d V_o}{0.1 P_o f_{sw}} = \underline{\underline{12 \mu H}}$$

c) When switch on,  $i_{pnode} = 0$ ,  $i_c = -i_o = -I_o$   
 When switch off,  $i_{pnode} = I_L$ ,  $i_c = i_{pnode} - i_o = I_L - I_o$   
 $I_L = 3.333 \text{ A}$ ,  $\Delta I_L = 0.1 I_o = 0.2 \text{ A}$   
 $I_o = P_o / V_o = 2 \text{ A}$



d)  $\Delta V_o = 0.01 V_o = 0.01(20) = \underline{0.2 \text{ V}}$

$\Delta V_o = \frac{Q}{C} = \frac{I_o \cdot DT_s}{C} = \frac{I_o \cdot D}{f_{sw} \cdot C} \leq 0.01 V_o$ ,  $I_o = P_o / V_o$

$\Rightarrow C \geq \frac{P_o D}{0.01 V_o^2 f_{sw}} = 0.2 \text{ mF}$

$\Rightarrow C_{min} = \underline{\underline{0.2 \text{ mF}}}$

$$N_1 : N_2 : N_3 = 1 : 2 : 1$$

$$V_d = 20 \text{ V}$$

$$f_{sw} = 20 \text{ kHz}$$

$$D = 0.2$$

$$L_m = 100 \mu\text{H}$$

$$R_{load} = 30 \Omega$$

$$V_o = ? \quad V_{sw}, i_d \text{ \& } i_D$$

Assume Converter is in CCM

$$\Rightarrow V_o = \frac{D}{1-D} V_d = \frac{0.2}{1-0.2} \cdot 20 \text{ V} = \underline{5 \text{ V}} \quad (\text{if derived good!!})$$

$$\Rightarrow I_o = \frac{V_o}{R_{load}} = \frac{5}{30} \text{ A} = \frac{1}{6} \text{ A} = \underline{0.167 \text{ A}}$$

When the switch  $S$  is off,  $D$  conducts  $\Rightarrow I_D = I_o$

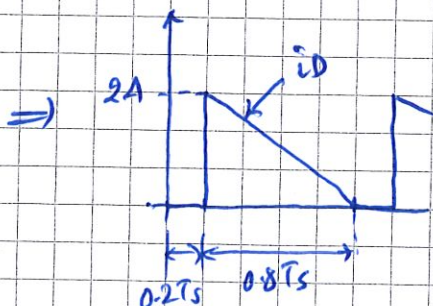
$$\text{But } I_o = I_D = \frac{V_o}{R_{load}} = 0.167 \text{ A}$$

During off period, the magnetizing current decreases proportional to the diode current.

For a Continuous Conduction mode we have

$$\frac{L \hat{\Delta} i_m}{(1-D)T_s} = +V_o \Rightarrow \hat{\Delta} i_m = \frac{(1-D)V_o}{L f_{sw}} \quad (2)$$

$$\Rightarrow \hat{\Delta} i_D = \frac{N_1}{N_3} \hat{\Delta} i_m = \frac{1}{1} \cdot \frac{(1-D)V_o}{L f_{sw}} = \underline{2 \text{ A}}$$



$$\Rightarrow I_{D, \text{min}} = \frac{1/2 (\hat{\Delta} i_D) (1-D) T_s}{T_s} = 0.8 \text{ A}$$

(for CCM)

But  $I_o = I_D = 0.167 < I_{D, \text{min}} = 0.8 \text{ A} \Rightarrow$  Converter in DCM

$\therefore$  For DCM,  $\frac{V_o}{V_d} = D \sqrt{\frac{2L_s}{2L_m}}$  (from energy conservation)

(the expression should be derived!!)

$\Rightarrow V_o = 0.2 \sqrt{\frac{30}{2(100\mu)(20k)}} \cdot 20V = \underline{\underline{10.9545V}}$

(2)

However due to the turns ratio,  $V_{o,max} = \frac{N_1 V_d}{N_2} = \underline{\underline{10V}}$

$\Rightarrow V_o = 10V$ , the secondary winding  $N_2$  limits the output voltage

For switch on period,  $i_d = i_m, v_{sw} = 0, i_o = 0$

For switch off period,  $D_2$  conducts due to the fact that  $V_o = V_{o,max}$

$v_{sw} = V_d + \frac{N_1 V_d}{N_2} = \underline{\underline{30V}} \Rightarrow i_D = 0$  but  $i_{D2} = \frac{N_1}{N_2} i_m = i_m/2$  &  $i_d = -i_{D2}$

For the discontinuous period,  $i_m = i_{D2} = i_d = i_o = 0, v_{sw} = V_d$

the interval the current goes off is  $\Delta t = \frac{L \Delta i_m}{V_d} \Rightarrow \frac{N_1 V_d}{N_2} = \frac{L \Delta i_m}{\Delta t \cdot T_s}$

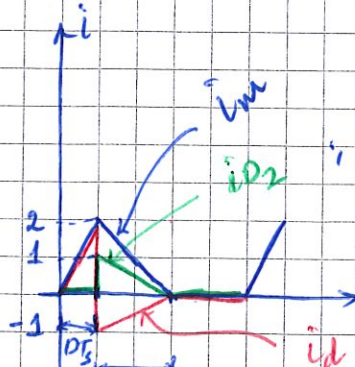
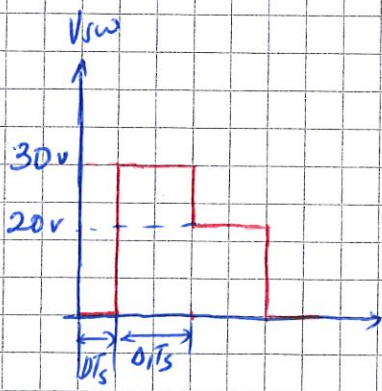
but  $V_d = \frac{L \Delta i_m}{\Delta t \cdot T_s}$  (during switch on)

$\Rightarrow \Delta t = 2D = \underline{\underline{0.4}}$

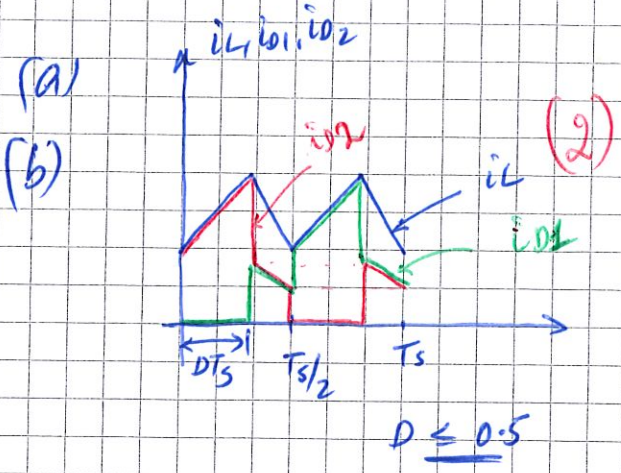
$i_D$  is only a transient current to account for capacitor loss and charge the voltage to 10V.

$\Delta i_m = \frac{V_d \cdot \Delta t \cdot T_s}{L} = 2A$

(2)



$i_D = 0$  all the time in steady state

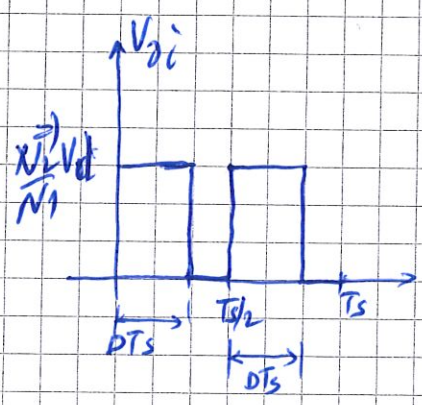


$$i_L = i_{D1} + i_{D2}$$

$$\left. \begin{matrix} T_1, T_2, \text{on} \\ T_3, T_4, \text{off} \end{matrix} \right\} \Rightarrow \begin{matrix} i_{D2} = i_L, i_{D1} = 0 \\ V_{oi} = \frac{N_2}{N_1} V_d \end{matrix}$$

$$\left. \begin{matrix} T_1, T_2, T_3, T_4, \text{off} \end{matrix} \right\} \Rightarrow \begin{matrix} i_{D2} = i_{D1} = i_L/2 \\ V_{oi} = 0 \end{matrix}$$

$$\left. \begin{matrix} T_1, T_2, \text{off} \\ T_3, T_4, \text{on} \end{matrix} \right\} \Rightarrow \begin{matrix} i_{D1} = i_L, i_{D2} = 0 \\ V_{oi} = V_d \cdot \frac{N_2}{N_1} \end{matrix}$$



$$\Rightarrow V_o = V_{oi} = \frac{1}{T_s} \left\{ 2 * DT_s * \frac{N_2}{N_1} V_d \right\} = \frac{2N_2 D}{N_1} V_d$$

$$\Rightarrow \frac{V_o}{V_d} = \frac{2N_2 D}{N_1} \quad (2)$$

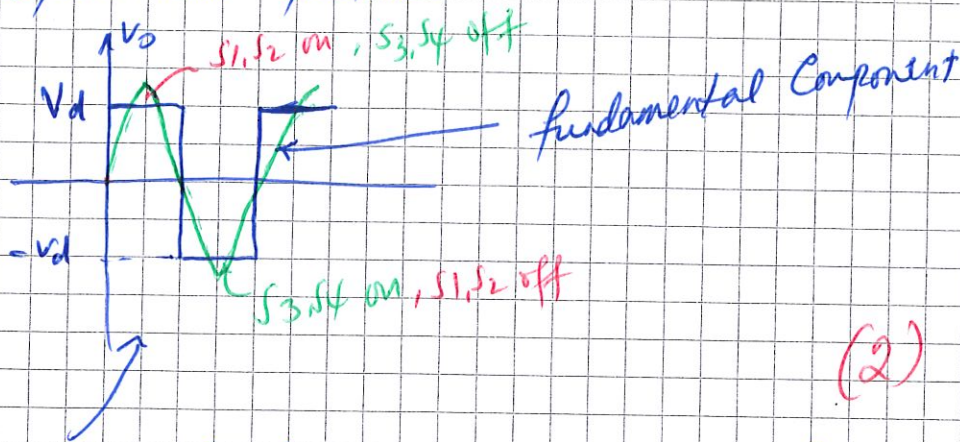
+ , capacitor at the dc bus optional unlike half-bridge

(c) + , twice the output voltage of a half bridge  
 ⇒ higher power rating  
 ⇒ possible to use less windings for the same  $V_o$

- , more Component Count (2)
- , more conduction loss ⇒ less economical for lower power rating

$V_d = 300V$

(a) Square wave output



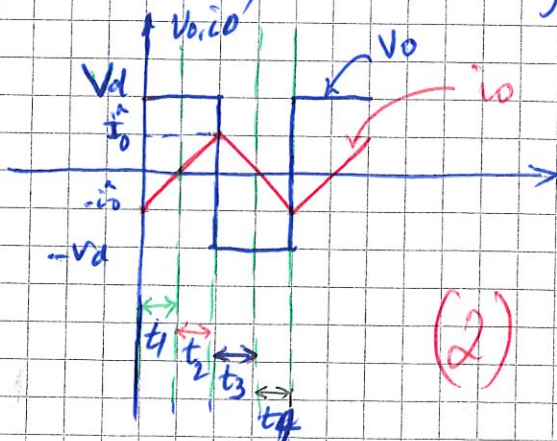
odd quarter wave  $\Rightarrow a_n = 0$  for all  $n$   
 $b_n = \frac{4}{\pi} V_d$  for  $n$  odd

$\Rightarrow b_1 = \frac{4}{\pi} V_d = \underline{\underline{382V}}$

$\Rightarrow \hat{V}_{oc(1)} = 382V, V_{oc(1) RMS} = \underline{\underline{270V}}$

(b)  $V_o = L \frac{di}{dt} \Rightarrow i_L = \frac{1}{L} \int_0^t V_o(t) dt$ ; being in  $A_c, i_L = 0$

$\Rightarrow i_L$  increases & decreases from an average value of 0.  
 Since  $V_o$  is square wave,  $i_L$  increases/decreases linearly when  $V_o$  is positive and negative respectively.

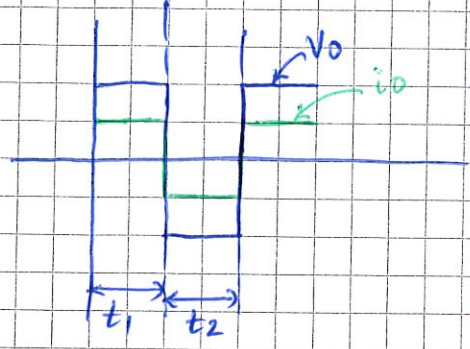


4 Quadrant operation possible

time	Device conducting
$t_1$	D1, D2
$t_2$	S1, S2
$t_3$	S3, S4
$t_4$	D3, D4

(c) For a purely resistive load

$V_o = Ri_o \Rightarrow i_o = V_o/R$



(2)

time	Device Conducting
t1	S1, S2
t2	S3, S4

(d)  $\hat{V}_{oc(1)} = m_a \cdot V_d \Rightarrow \hat{V}_{oc(1)} = 1 \cdot 300 = \underline{\underline{300V}}$   
 ( $m_a \leq 1$ )

$V_{oc(1)_{rms}} = \frac{300}{\sqrt{2}} = \underline{\underline{212V}}$

(2)

(e) square wave

- |  |   |   |
|--|---|---|
| +  | + | -   |
| <ul style="list-style-type: none"> <li>✓ smaller switching loss</li> <li>✓ Single modulation</li> <li>✓ higher fundamental output</li> </ul> |   | <ul style="list-style-type: none"> <li>✓ higher low order harmonics</li> <li>✓ uncontrolled output voltage</li> </ul> |

PWM operation

- |  |   |   |
|--|---|---|
| +  | + | -   |
| <ul style="list-style-type: none"> <li>✓ smaller low order harmonics</li> <li>✓ controlled output voltage</li> </ul> |   | <ul style="list-style-type: none"> <li>✓ higher switching loss</li> <li>✓ complex control</li> <li>✓ electromagnetic disturbance</li> </ul> |

(f) three-phase :-

- |  |
|--|
| +  |
| <ul style="list-style-type: none"> <li>✓ low harmonics</li> <li>✓ higher power output</li> </ul> |

- |   |
|---|
| -   |
| <ul style="list-style-type: none"> <li>✓ more component count <math>\Rightarrow</math> expensive</li> <li>✓ more losses <math>\Rightarrow</math> less economical for lower power output &amp; single phase loads</li> </ul> |

(2)



(a)  $V_{LL} = 300V$ ,  $I_d = 10A$ ,  $50Hz$

current flows in phase a when  $V_a$  is higher or lower than both  $V_b$  and  $V_c$ .

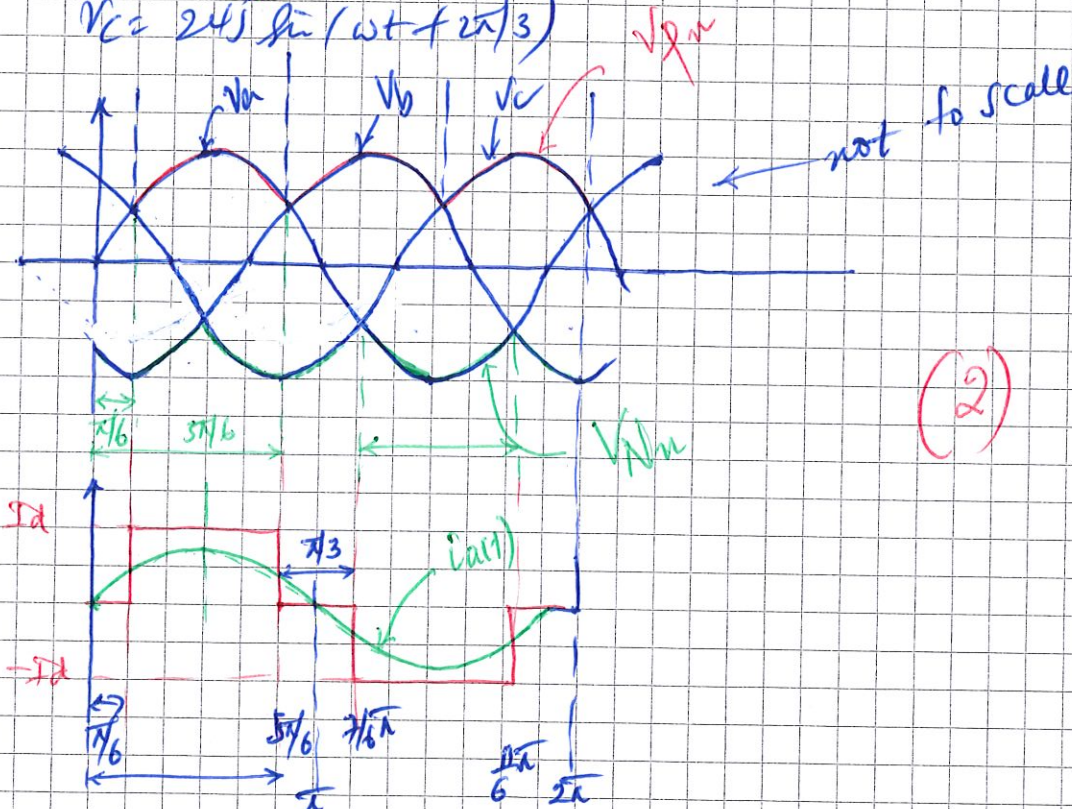
the output voltage  $V_d = \max(V_a, V_b, V_c) - \min(V_a, V_b, V_c) = V_{p1} - V_{n1}$

$300 LL \Rightarrow V_a = \frac{300\sqrt{2}}{\sqrt{3}} = 245V$

$\Rightarrow V_a = 245 \sin(\omega t)$

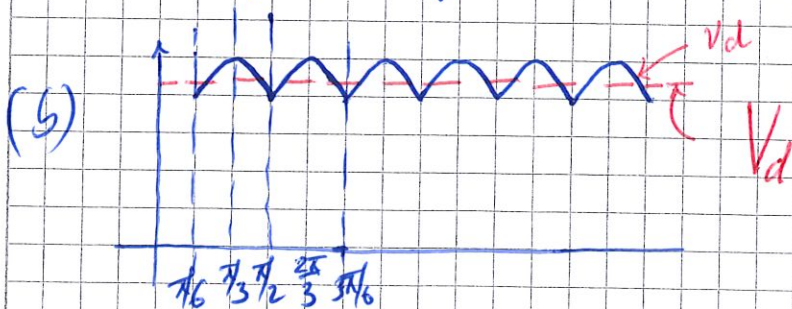
$V_b = 245 \sin(\omega t - 2\pi/3)$

$V_c = 245 \sin(\omega t + 2\pi/3)$



(2)

No phase shift between  $V_a$  &  $i(a)$   $\Rightarrow$  DPF = 1



for one cycle,  $\pi/6$  to  $\pi/2$ ,  $v_d = (V_{LL}\sqrt{2}) \cos(\theta - \pi/3)$

$$\Rightarrow V_d = \frac{1}{\pi/3} \int_{\pi/6}^{\pi/2} V_{LL}\sqrt{2} \cos(\theta - \pi/3) d\theta \quad (2)$$

$$= \frac{3}{\pi} \int_{\pi/6}^{\pi/2} V_{LL}\sqrt{2} \cos\theta \cdot d\theta = \frac{3\sqrt{2} V_{LL} \sin\theta}{\pi} \Big|_{\pi/6}^{\pi/2} = \frac{3\sqrt{2} V_{LL}}{\pi} = \underline{\underline{405V}}$$

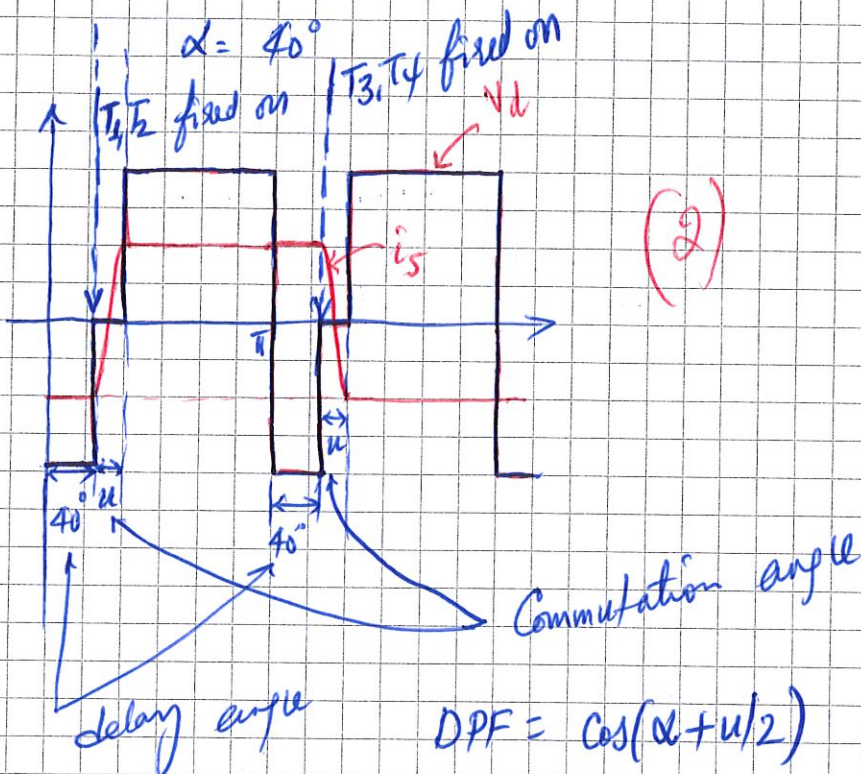
(c) Source inductance adds a commutative angle,  $\mu$ .

$\Rightarrow \text{DPF} = \cos(\mu/2) \Rightarrow$  decreases (2)  
 this also reduces the average output voltage.

(a)  $L_s = 5\text{mH}$ ,  $50\text{Hz}$  system

$$i_d = I_d = 20\text{A}$$

$$V_s = \pm 150\text{V}$$



(b) during 2 commutations,  $V_d = 0$  ( $2 \times u$ )  
 during  $2 \times 40^\circ$ ,  $V_d = -150\text{V}$  ( $2 \times 40^\circ$ )  
 for other intervals,  $V_d = 150\text{V}$  ( $360^\circ - 2 \times 40^\circ - 2 \times u$ )

$$\Rightarrow V_d = \frac{(360 - 2 \times 40^\circ - 2 \times u) 150 - (2 \times 40^\circ) 150 + (2 \times u) 0}{360}$$

$$\Rightarrow V_d = \frac{360 - 160 - 2u}{360} \cdot 150, \quad u \text{ in degrees}$$

find  $u$ ?

$i_s$  increases from  $-I_d$  to  $I_d$  in interval of  $\mu$

$$\Rightarrow V_s = L_s \frac{di_s}{dt} \quad \text{where } V_s = 150V$$

$$\Rightarrow V_s = L_s \frac{di_s}{d\theta} \cdot \frac{d\theta}{dt} = L_s \omega \cdot \frac{di_s}{d\theta} \quad (2)$$

$$\Rightarrow \int_0^{\mu} V_s d\theta = L_s \omega \int_{-I_d}^{I_d} di_s = 2L_s \omega I_d$$

$$\Rightarrow V_s \cdot \mu = 150\mu = 2L_s \omega I_d$$

$$\Rightarrow \mu = \frac{2L_s \omega I_d}{150} \text{ rad} = \frac{2L_s \omega I_d}{150} \cdot \frac{180}{\pi} \text{ degrees} = \underline{\underline{24^\circ}}$$

$$\Rightarrow V_d = \frac{360 - 160 - 2 \times 24}{360} \cdot 150 = \underline{\underline{63.3V}}$$

(c) the source inductance results in  $V_d$  during the commutation angle.  $\mu$  results in a reduction of the average voltage by

$$\frac{2 \times 24}{360} \cdot 150V = \underline{\underline{20V}} \quad (2)$$