Examination **ENM060 Power Electronic Converters**

Date and time Tuesday August 23rd, 2016, 14:00 – 18:00

Responsible Teacher: Andreas Henriksson, tel. 0709-524924

Authorised Aids: Chalmers-approved calculator (Casio FX82..., Texas Instruments Ti-30... and Sharp EL-W531...)

Grades: U, 3, 4 or 5. (The limit for a 3 on the exam is 20p, a 4 is 30p and 5 is 40p. The maximum number of points is 50.)

Solutions: Course webpage (Ping-Pong), August 24th 2016

Review of Exam September 15th and September 19th, 12:00-13:00.
Uno Lamms Room. Division of Electric Power Engineering (2nd floor).

From September 20th 2016, the exams can be picked-up at the exam office, Department of Energy and Environment.
Location: EDIT building, Maskingränd 2, 3Ö (east) floor, room 3434A.
Opening hours during semesters: Monday-Friday 12:30-14:30

Observe that the questions are not arranged in any kind of order.

On the last pages there are some formulas that can be used in the examination. Always assume steady-state conditions in all tasks unless otherwise stated.

Please, read through the exam before you start.

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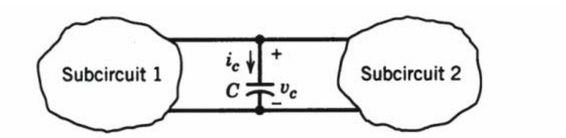
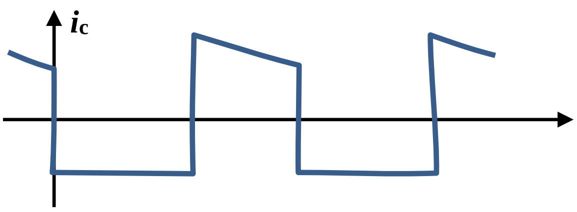
1) The voltage curve-form (v_c) is applied over the capacitor which are connected to two arbitrary sub-circuits. (3p)

- **Explain the concept of steady state.**
- **Sketch the resulting current that flows into the capacitor (i_c).**



The capacitor current equals the derivative of the capacitor voltage. If the capacitor is in steady state, no net storage of charges occur over one period. This means that both the voltage and the current has the same value in the beginning and at the end of a switching period.



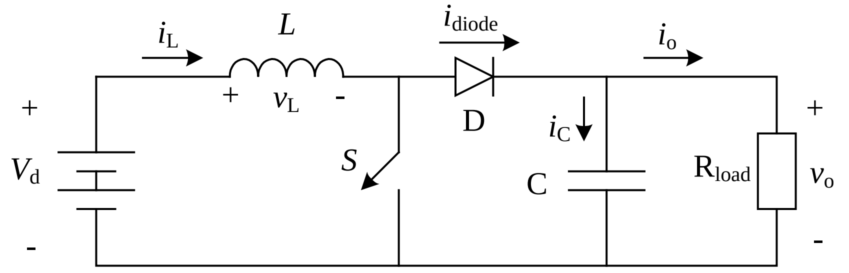
$$i_c = C\frac{dv_c}{dt}$$

2) Consider the ideal boost converter below.

- **Calculate the peak-to-peak output voltage ripple for the specified operating point.**
- **Draw the curve form of the voltage ripple. (5p)**

$\boldsymbol{V_d = 8V}$ $\quad\quad$ $\boldsymbol{V_o = 12V}$ $\quad\quad$ $\boldsymbol{I_o = 6A}$

$\boldsymbol{f_{sw} = 320kHz}$ $\quad\quad$ $\boldsymbol{C = 270\mu F}$ $\quad\quad$ $\boldsymbol{L = 1.4\mu H}$



We start by assuming that the converter is operating in steady-state and in CCM and that the output capacitor is so large that the output voltage can be treated as a DC voltage. First we have to find an expression for the duty ratio. We do this by calculating the average inductor voltage.

$$V_L = \frac{1}{T_{sw}}\int_0^{T_{sw}} v_L dt = \frac{1}{T_{sw}}(V_d D T_{sw} + (V_d - V_o)(1-D)T_{sw}) = V_d + V_o(1-D) = 0$$

$$V_o = \frac{V_d}{1-D} \quad \rightarrow \quad D = \frac{V_o - V_d}{V_o} = \frac{12V - 8V}{12V} = 0.33$$

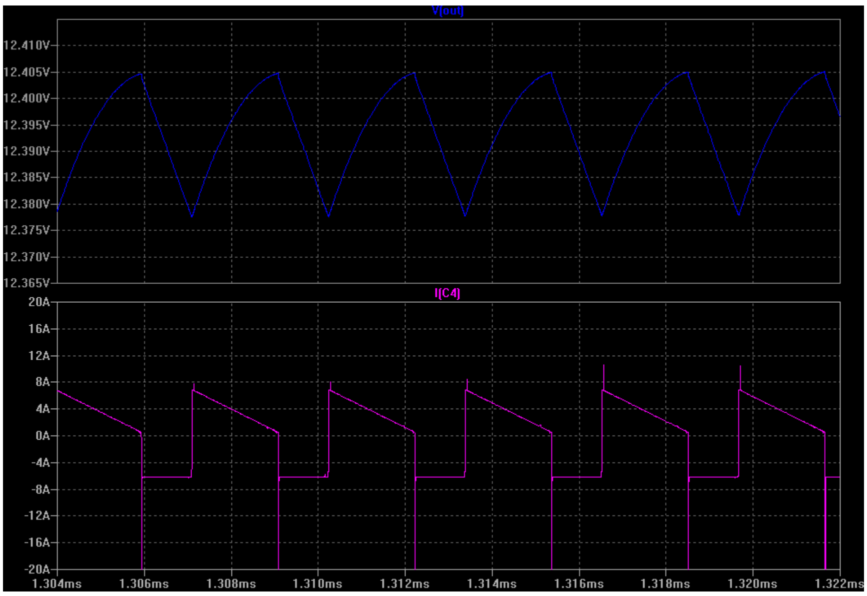
The average input current we calculate by knowing that we have a loss less converter consisting of only ideal components.

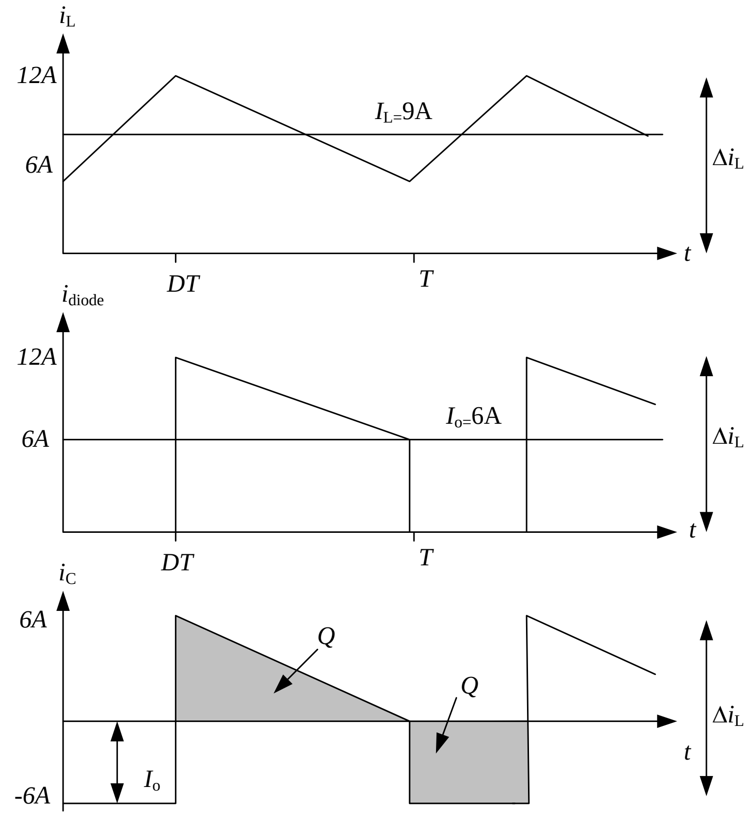
$$P_d = P_o \quad \rightarrow \quad I_L = \frac{V_o I_o}{V_d} = \frac{V_o I_o}{V_o(1-D)} = \frac{I_o}{(1-D)} = \frac{6A}{(1-0.33)} = 9A$$

The assumption of CCM has to be checked. The peak-to-peak ripple in the inductor current we calculate by studying the inductor voltage during the on-time of the switch.

$$\Delta i_L = \frac{V_d DT}{L} = \frac{V_d D}{L f_{sw}} = \frac{8V \cdot 0.333}{1.4\mu H \cdot 320kHz} = 5.95A$$

The converter is operating in CCM. Now we plot the inductor current, the diode current and the capacitor current. The output current is a DC current, since we have assumed a constant output voltage.





$$i_D = i_C + I_o$$
$$i_C = i_D - I_o$$

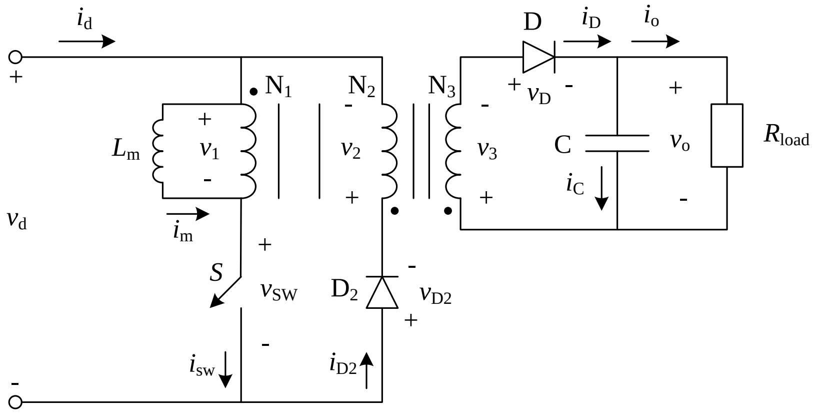
The output ripple can be expressed as:

$$\Delta v_o = \frac{Q}{C} = \frac{1}{C} \cdot I_o DT$$

$$\Delta v_o = \frac{6A \cdot 0.33 \cdot {}^{1}/_{320kHz}}{270uF} = 23mV$$

3) Consider the Flyback converter below. (5p)

- **Check if the converter is operating in CCM or DCM.**
- **Calculate the resulting temperature of the output diode (*D*) if the ambient temperature is 30°C. Application of Simpsons formula must be used for full points.**



$$\boldsymbol{L_m = 120\mu H}$$
$$\boldsymbol{C = 330\mu F}$$
$$\boldsymbol{V_d = 20V}$$
$$\boldsymbol{V_o = 32V}$$
$$\boldsymbol{f_{sw} = 250kHz}$$
$$\boldsymbol{R = 4\Omega}$$
$$\boldsymbol{(N_1 : N_2 : N_3) = (1 : 0.5 : 1)}$$
$$\boldsymbol{V_f = 0.84V}$$
$$\boldsymbol{R_{\theta ja} = 10^\circ C/W}$$

The ratio between the input and output voltage is:

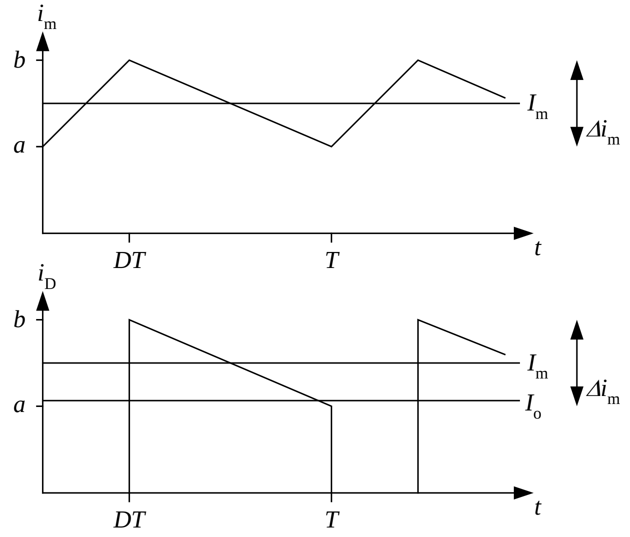
$$V_L = \frac{1}{T_{sw}} \int_0^{DT_{sw}} V_d dt + \frac{1}{T_{sw}} \int_{DT_{sw}}^{T_{sw}} -V_o dt = \frac{1}{T_{sw}} (V_d DT_{sw} - V_o T_{sw} + V_o DT_{sw}) = 0$$

$$V_d D = V_o(1-D) \quad \rightarrow \quad \frac{V_o}{V_d} = \frac{D}{1-D} \quad \rightarrow \quad \frac{32V}{20V} = \frac{D}{1-D} \quad \rightarrow \quad D = 0.615$$

If the converter is operating in CCM, the current ripple must be smaller than twice the average magnetizing current ($\Delta i_m = 2I_m$). The current ripple in the magnetizing current can be expressed as:

$$\Delta i_m = \frac{V_d DT_{sw}}{L_m} = \frac{V_d D}{L_m f_{sw}} = \frac{20V \cdot 0.615}{120\mu H \cdot 250kHz} = 0.41A$$

The magnetizing current (i_m) and the diode current (i_D) for CCM can be drawn as:



From the figure we see that the average magnetizing current can be calculated as:

$$I_m = \frac{1}{T_{sw}}\int_0^{T_{sw}} i_m(t)dt = \frac{1}{T_{sw}}\int_0^{DT_{sw}} i_m(t)dt + \frac{1}{T_{sw}}\int_{DT_{sw}}^{T_{sw}} i_m(t)dt = \frac{a+b}{2T}DT + \frac{a+b}{2T}(1-D)T = \frac{a+b}{2}$$

$$I_o = \frac{1}{T_{sw}}\int_0^{T_{sw}} i_D(t)dt = \frac{1}{T_{sw}}\int_{DT_{sw}}^{T_{sw}} i_m(t)dt = \frac{a+b}{2T}(1-D)T = I_m(1-D) \quad \rightarrow \quad I_m = \frac{8A}{1-0.615} = 20.8A$$

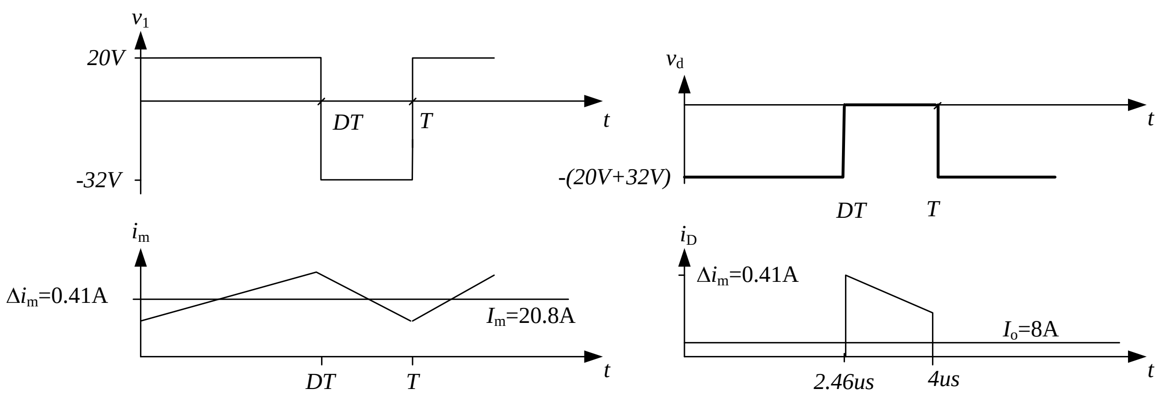
The border between CCM and DCM can now be calculated as:

$$\Delta i_m = 2I_m \quad \rightarrow \quad \frac{V_d D}{L_m f_{sw}} = \frac{2I_o}{(1-D)}$$

If solving for D and considering that for CCM, the current ripple has to be smaller than twice the average magnetizing current, the final answer is obtained:

$$D(1-D) < \frac{2I_o L_m f_{sw}}{V_d} \quad 0.615 \cdot (1-0.615) < \frac{2 \cdot 8A \cdot 120\mu H \cdot 250kHz}{20V}$$

Which means that the converter is operating in CCM. The diode voltage and current can now be drawn:



To calculate the power loss in the diode, only the constant voltage drop is accounted for, this gives a need to calculate the average current through the diode. The ripple current through the diode is small compared to the total current (0.6A compared to 20.8A and 8A) and can be neglected in the calculation of the average current.

$$I_{diode(AVG)} = \frac{1}{4\mu}\int_0^{4\mu} i_{diode}(t)dt$$

The definition for Simpsons rule is:

$$\frac{1}{T}\int_{t_0}^{t_0+T} f(x)dx = \frac{1}{6}\left(f(t_0) + 4\cdot f\left(t_0 + \frac{T}{2}\right) + f(t_0+T)\right)$$

The AVG current can now be calculated as:

$$I_{diode(AVG)} = \frac{1}{4\mu}\left(2.46\mu\frac{1}{2.46\mu}\int_0^{2.46\mu} 0dt + 1.54\mu\frac{1}{1.54\mu}\int_{2.46\mu}^{4\mu} i_{diode}(t)dt\right) =$$

$$= \frac{1.54\mu}{4\mu}\frac{1}{6}\big((21.0A) + 4(20.8A) + (20.6A)\big) = 8A$$

Both diodes can handle the current which gives that the power dissipation can be calculated as

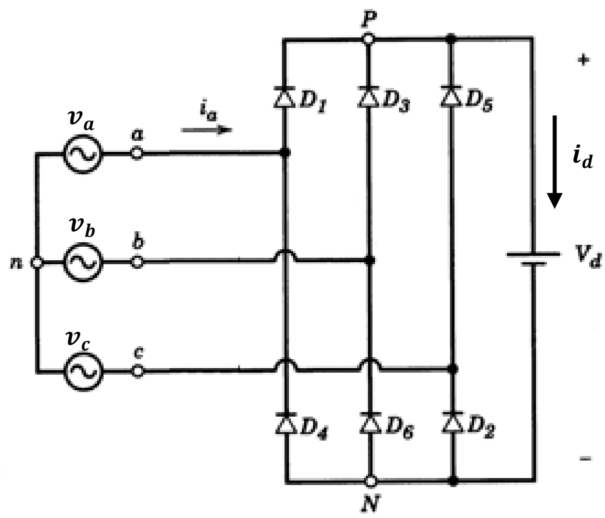
$$P_{conduction} = V_F I_{diode(AVG)} = 0.84V\cdot 8A = 6.7W$$

The resulting component temperature can be calculated as:

$T_j = R_{\theta ja}\cdot P_{conduction} + T_a = 10°\,C/W\cdot 6.7W + 30°C = 97°C$ D

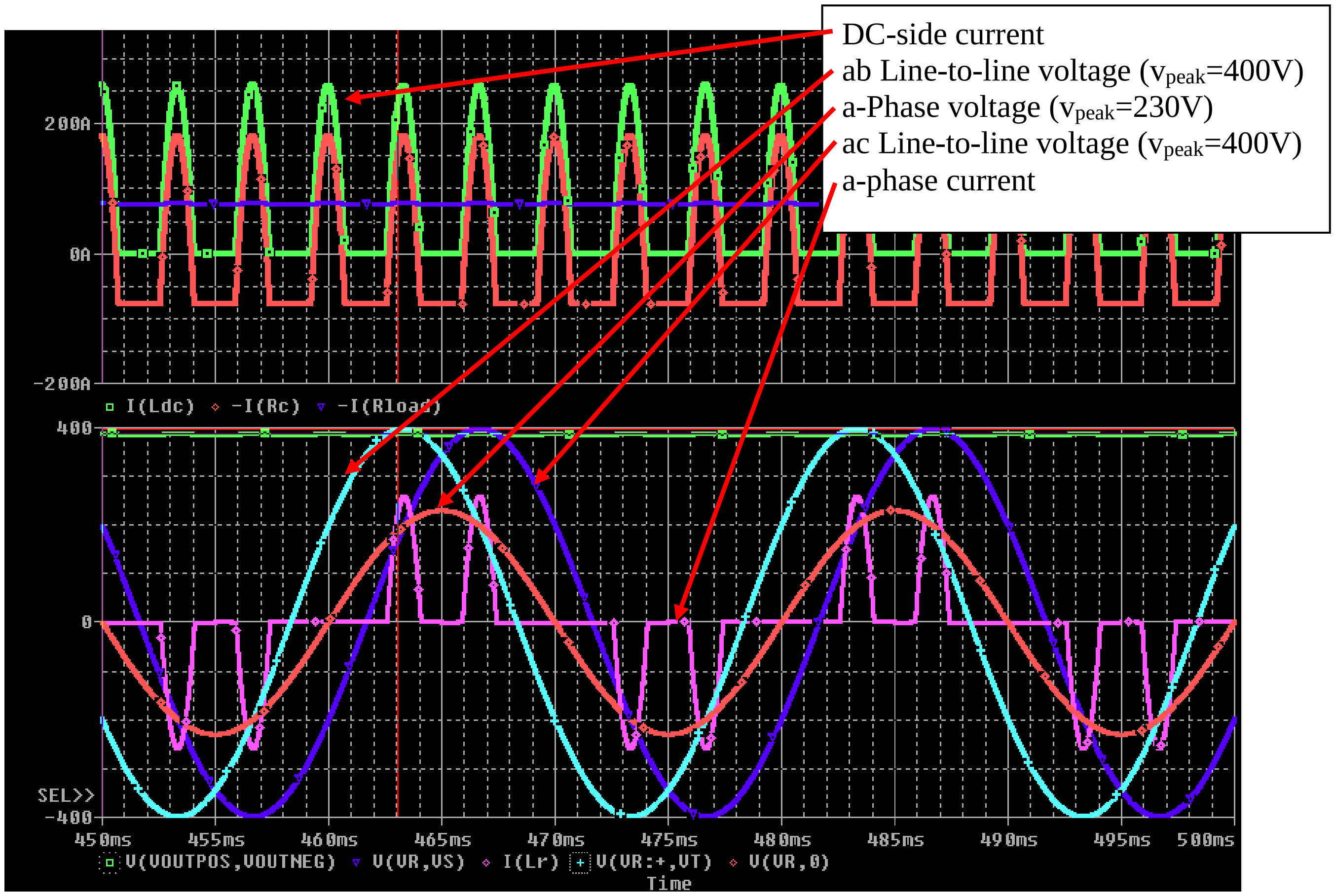
4) The three phase diode rectifier depicted below is used with a voltage stiff DC-link. The system operates with $50Hz$ and $v_a = v_b = v_c = 230V$ peak voltage. Draw the voltages and currents stated below. Clearly state the amplitudes and phase shifts between the voltages, exact amplitudes of the currents are not needed. (5p)

- **The phase voltage in phase a (v_{an})**
- **The line-to-line voltages for phase a (v_{ab} and v_{ac})**
- **The current in phase a (i_a).**
- **The DC-side current (i_d).**



Current is drawn from the voltage source as long as the input voltage is greater than the output voltage. Note that this happens two times for each phase, when the line-to-line voltages (v_{ab} and v_{ac}) are greater than the output voltage, a current is drawn from the source. This can be seen as two pulses in the middle of the a-phase voltage.

Points are given if the answer shows a difference in amplitude and phase (30°) between the phase and line-to-line voltages. To score full points, the correct phase current has to be drawn.



5) A single-phase diode rectifier with a voltage stiff DC-side (large DC-link capacitor) is connected to a grid with a source inductance. (4p)

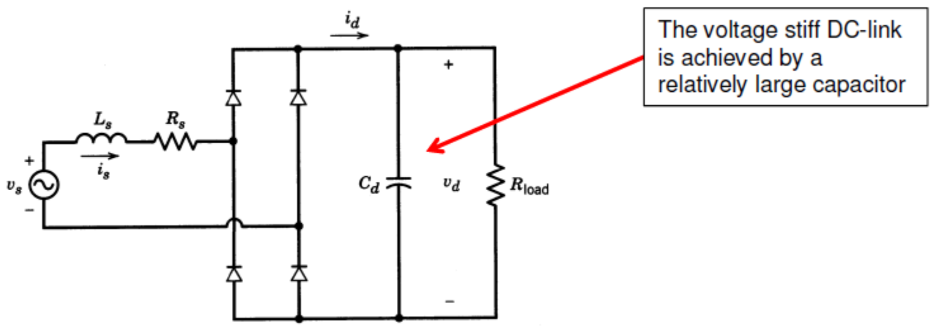
- **Sketch a typical waveform of the resulting line current (i_s)**
- **How will the source inductance affect the power factor (PF) and the displacement power factor (DPF)? Exemplify with e.g. graphs and Fourier components.**

If the source inductance is taken into account, the current waveforms will be smoothened out due to the inductance, see Lecture 12. As the source inductance continues to increase, the DC-side current will eventually become continuous.

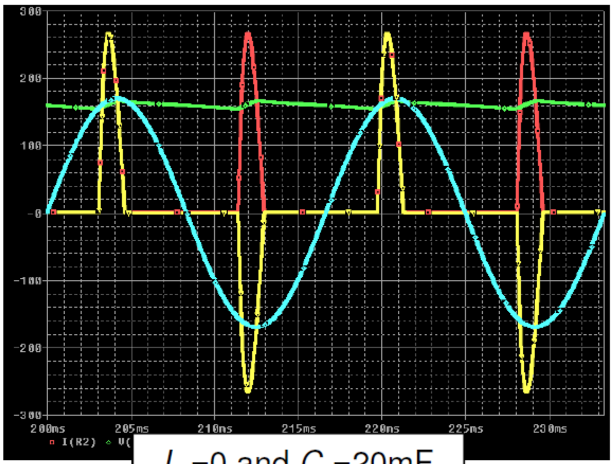
A large distortion in the current waveform will result in a low PF

$$PF = \frac{I_{S(1)}}{I_S}\cos(\varphi_1)$$

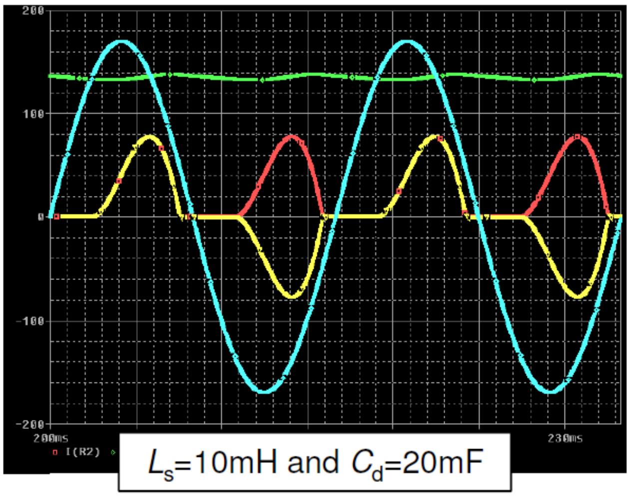
$$DPF = \cos(\varphi_1)$$



For the single phase diode rectifier with a source inductance, the fundamental frequency component of the line current will change (see Fig 5-18 in Undeland) if the source inductance is increases. If the phase difference between the fundamental frequency current component and the source voltage changes, the resulting DPF will decrease. The power factor (PF) will however increase since the harmonics decreases as the source inductance increases.



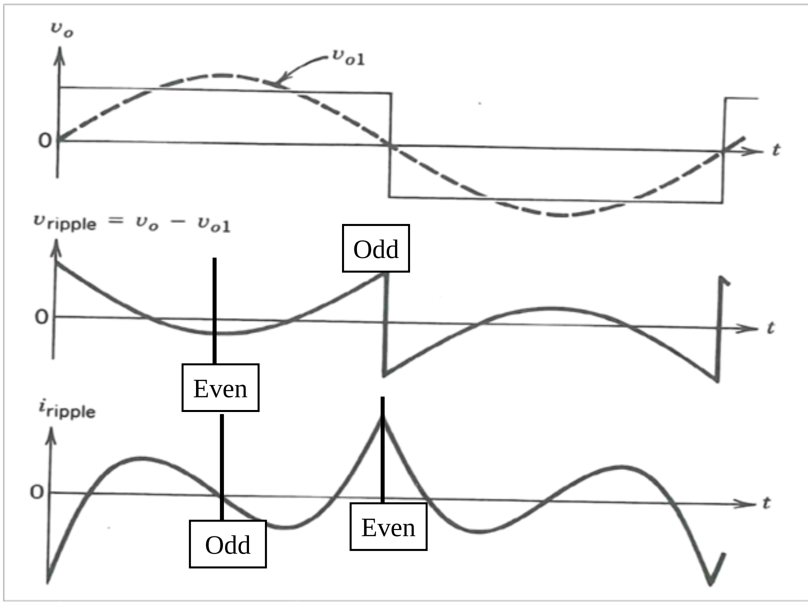
L_s=0 and C_d=20mF



L_s=10mH and C_d=20mF

6) For a single phase inverter operating in square wave mode, calculate the peak-to-peak ripple in the output current. Assume that the fundamental frequency component of the output voltage is $V_{o(1)} = 156V$ (50Hz) and that the load consists of an inductor $(L = 100mH)$ in series with a sinusoidally shaped back-emf voltage source $\left(e_o = \sqrt{2} \cdot E_o \cdot \sin(\omega_1 t)\right)$. The back-emf has the same aplitude, frequency and phase shift as the fundamental frequency component of the output voltage. The DC-link voltage (V_d) is 170V. (5p)

This task can be solved in an analogous way as for the three-phase inverter operating in square-wave mode.



The current is formed by the voltage applied over the load inductor. Due to the sinusoidal-shaped back-emf of the load, the ripple voltage will be applied over the inductor.

$$v_{o(ripple)} = v_o - v_{o(1)}$$

The ripple voltage is known since the output fundamental is $V_{o(1)} = 156V$ (RMS voltage) and the output voltage is a square wave with 170V amplitude.

Since the ripple voltage is even at π/2 and odd at π, the ripple current must be odd at π/2 and even at π. This gives that the current ripple can be calculated by integrating the ripple voltage from T/4 to T/2.

$$i_{o(ripple)}(\theta) = \underbrace{i_{o(ripple)}\left(\frac{\pi}{2}\right)}_{=0} + \frac{1}{\omega L}\int_{\pi/2}^{\pi} v_{o(ripple)}(\omega t)d\omega t = \frac{1}{\omega L}\int_{\pi/2}^{\pi}\left(V_d - \hat{V}_{o(1)}\sin(\omega t)\right)d\omega t =$$

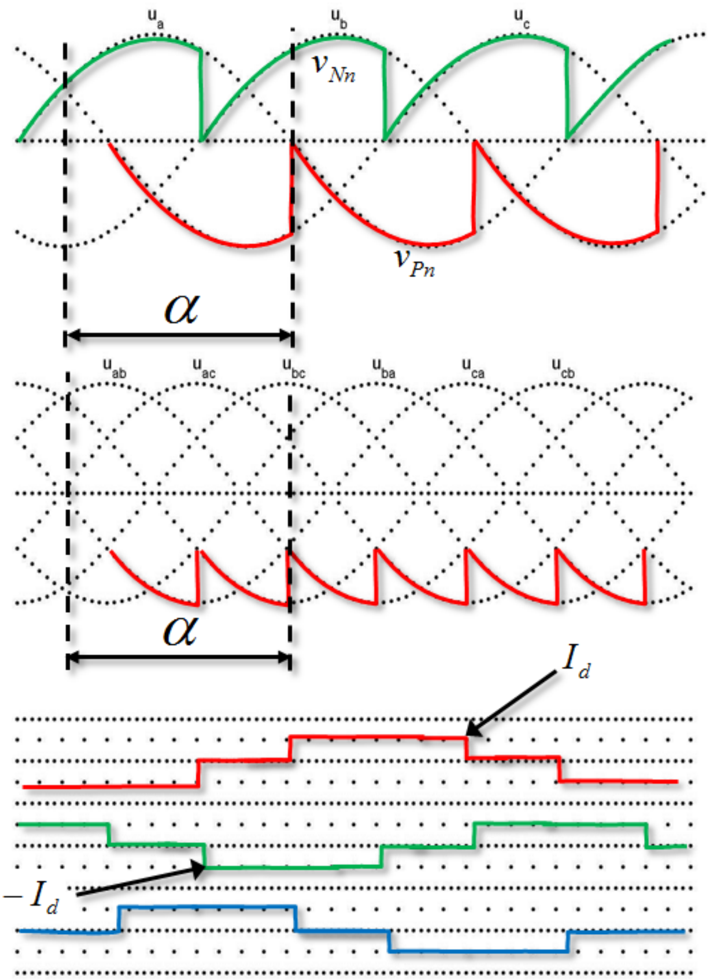
$$= \frac{1}{\omega L}\int_{\pi/2}^{\pi}\left(170V - \sqrt{2}\cdot 156V\cdot\sin(\omega t)\right)d\omega t = \frac{1}{\omega L}\left[170V\cdot\omega t + \sqrt{2}\cdot 156V\cdot\cos(\omega t)\right]_{\pi/2}^{\pi} =$$

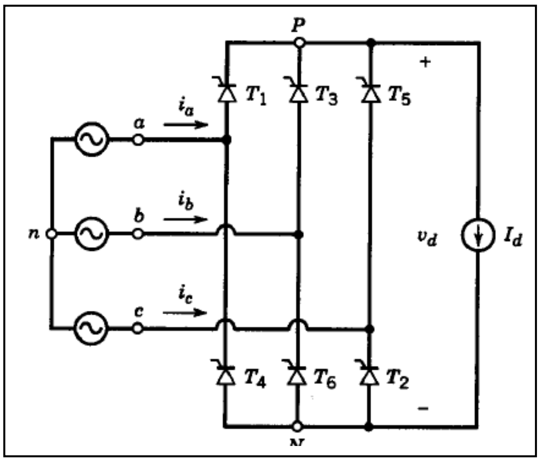
$$= \frac{1}{\omega L}\left(170V\cdot\pi + \sqrt{2}\cdot 156V\cdot(-1) - 170V\cdot\frac{\pi}{2} - \sqrt{2}\cdot 156V\cdot 0\right) = \frac{1}{\omega L}\left(170V\cdot\frac{\pi}{2} - \sqrt{2}\cdot 156V\right) \rightarrow$$

$$\Delta i_{o(ripple)} = 1.48A$$

7) A three-phase thyristor inverter is operating with a constant current load ($I_d = 25A$). The three-phase network is assumed be a balanced system with no line impedance. (4p)

- **Draw the voltage waveforms (v_{Nn}, v_{Pn}, v_d) for $\alpha = 150°$ on the enclosed dot-paper.**
- **Draw the current waveforms (i_a, i_b, i_c) and state the amplitude**



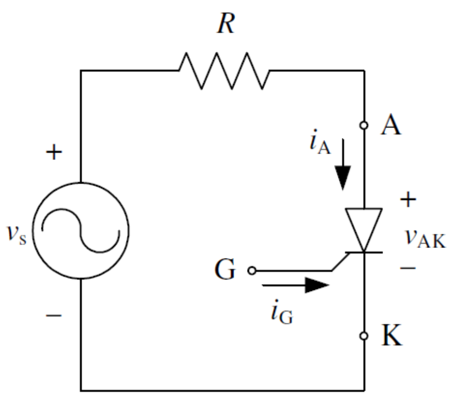


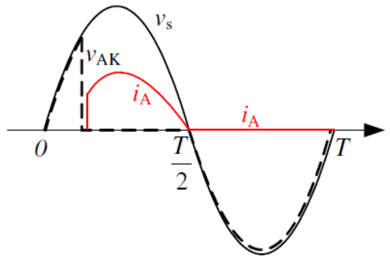
If a large firing angle is used, there is very little voltage-time area in order to commutate the current over to the next phase, i.e. that the commutation angle must be sufficiently small. Also, the extinction angle ($\gamma = 180° - \alpha - u$) must greater than the thyristor turn off interval, otherwise the tyhristor might continue to conduct, see page 149 in Undeland.

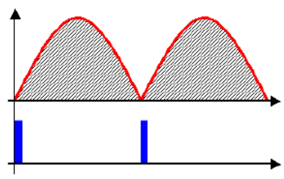
8) For the Thyristor inverter in (7), each component is turned on by applying a pulse on the gate terminal. (3p)

- **Is the thyristor a current or voltage controlled device?**
- **Explain why it could be a problem to operate this inverter with large firing angles (large α), for example 180°.**

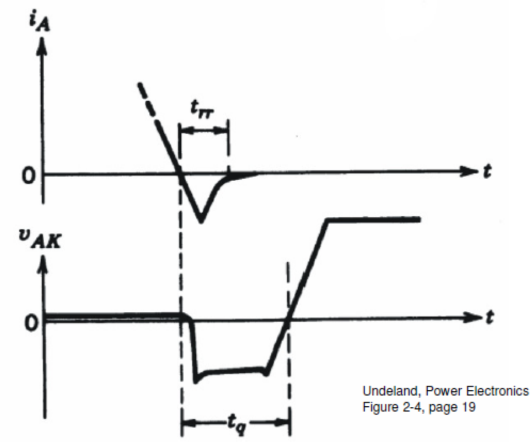
The Thyristor is a current controlled device that is turned on by applying a current pulse on the gate terminal.







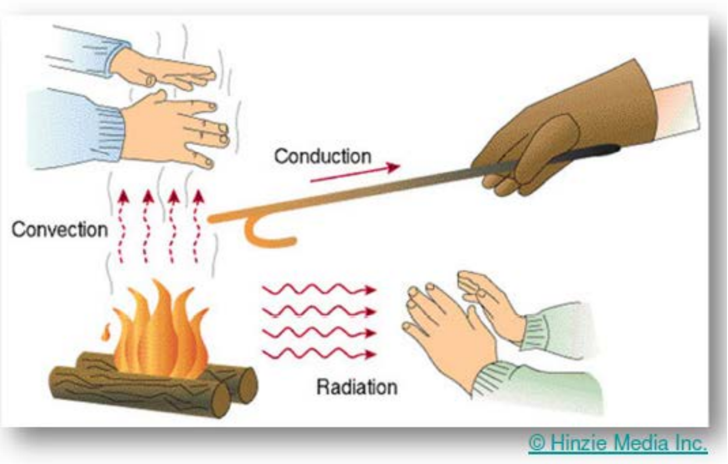
- After a thyristor has been switched off by forced commutation, a time delay (t_q) must elapsed before it can be positively biased again.
- If the time interval is too short, the tyristor can self-trigger by the remaining charge carriers that have not yet recombined.



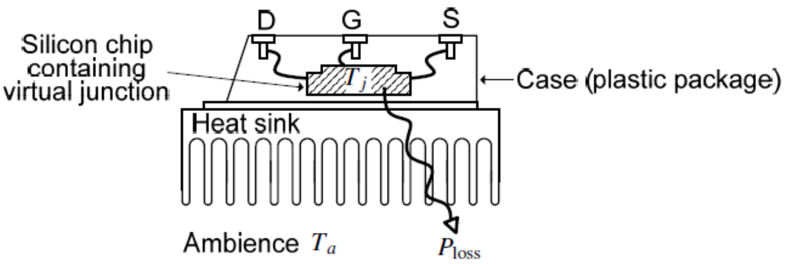
9) Heat can be transferred by three different mechanisms and it is important to design your application so that it does not become too hot. If a semiconductor becomes too hot and is exposed to extensive thermal cycling, it will eventually break down. (4p)

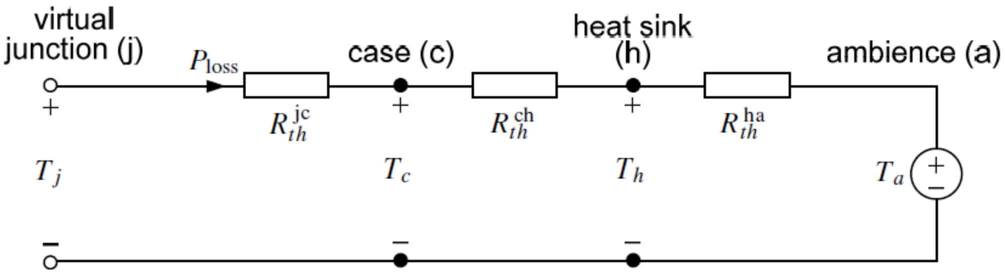
- **Which are the three heat transfer mechanisms and which one is the most important in semiconductors for power electronic applications?**
- **What is the typical reason for component failure in a semiconductor for power electronic applications?**

- Radiation
- Convection
- Conduction

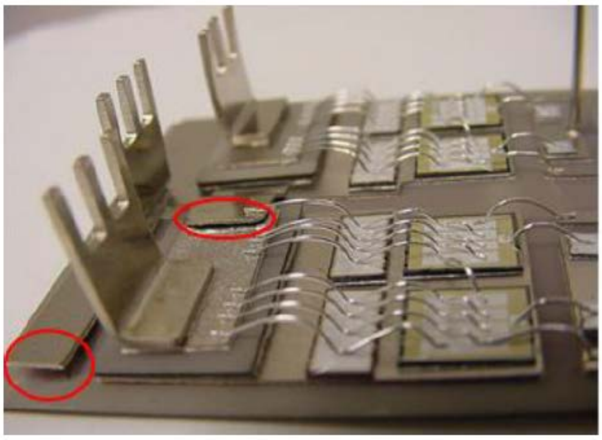


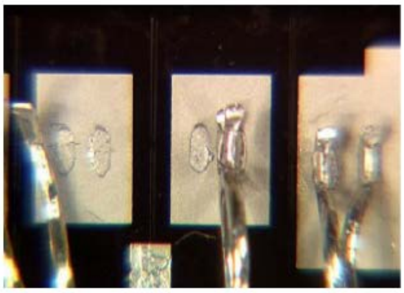
The most important heat transfer mechanism in power semiconductors is conduction which is accounted for in the thermal resistance that can be calculated for different materials. For the interface between the heat sink and the ambient, the convection will also play an important role since it affects the thermal resistance for this interface. High convection (or forced convection) will result in a low thermal resistance.





- For power modules, thermal cycling will give rise to material defects, typically in the solder joints or die attachments

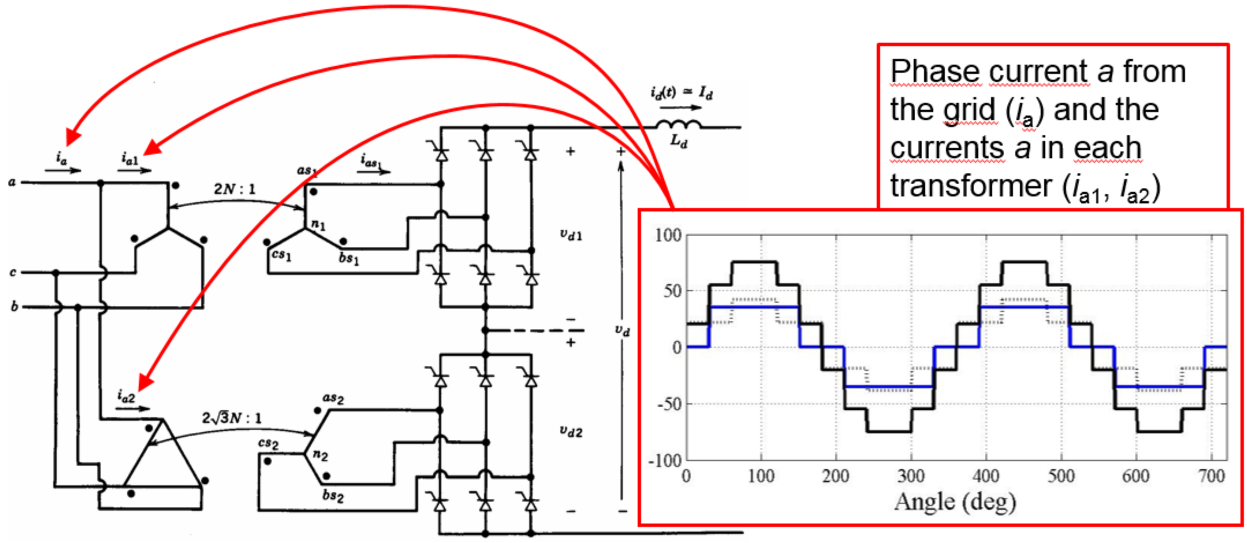




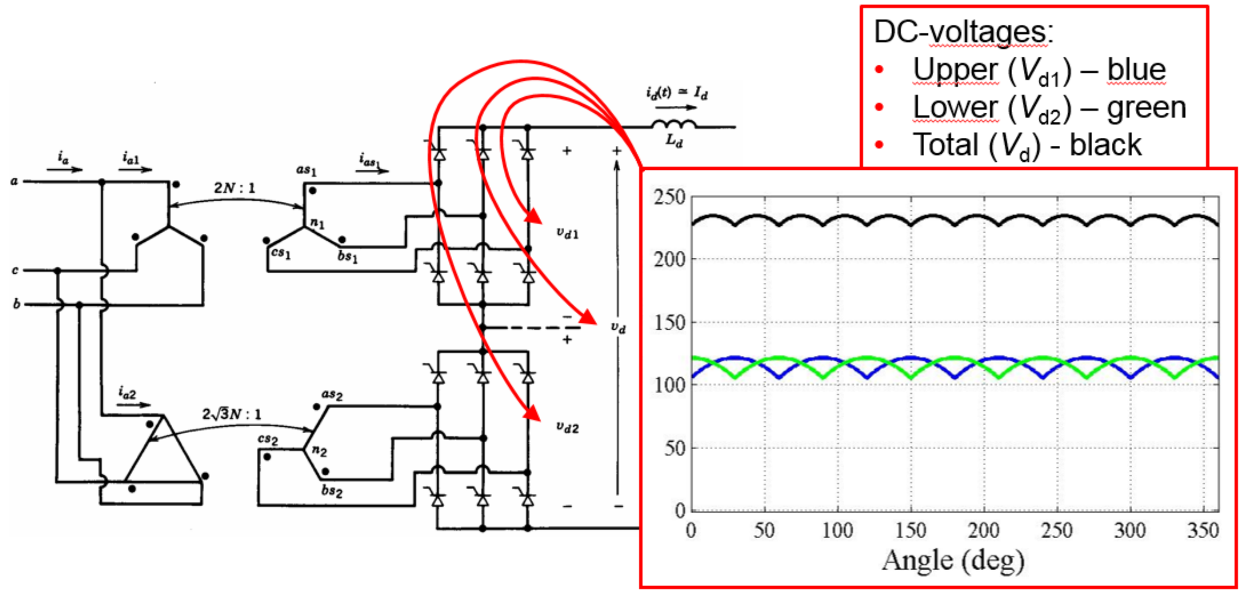
10) What are the main benefits of using 12-pulse rectification in a three-phase thyristor application (e.g. a HVDC classic)? Exemplify with e.g. input current and output voltage. (4p)

- To reduce line current THD
- To improve input power factor
- To avoid semiconductor devices in series.

For a 6-pulse rectifier, the THD in the line current is high and the PF is low due to harmonics. For a corresponding 12-pulse rectifier, the THD in the line current is significantly lower and the PF is almost unity.

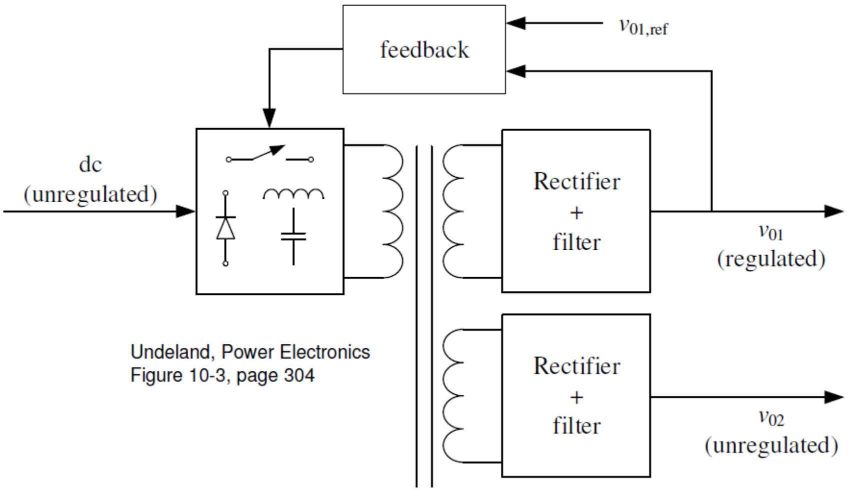


The resulting output DC-link voltage will be smoother (lower harmonic content) since it will consist of two voltages from each thyristor rectifier that are summed together with 30º phase difference.

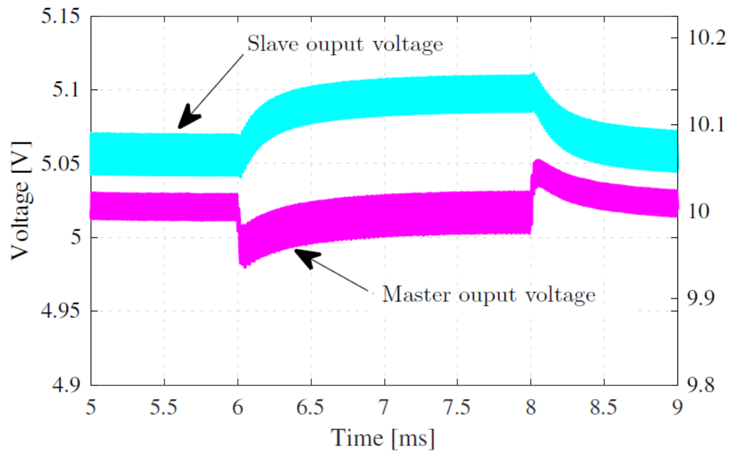


11) A Flyback converter with two secondary windings (i.e. two output voltages v_{01} and v_{02}) is used with real components including parasitic elements. The controller circuit only regulates one output voltage (v_{01}). Explain thoroughly what happens if a load step (e.g. sudden increase/decrease of the output current) is applied on output v_{01} and v_{02}, respectively. (4p)

The Flyback converter with multiple outputs can be drawn as:



If a load step (sudden current increase) is applied on the master output voltage (v_{01}) at t=2ms, the output voltage is suddenly decreased due to increased losses in the converter. This is detected by the controller which increases the duty cycle in order to keep the output voltage constant. The second output (v_{02}) is however not regulated and will therefore experience an increase due to the increased duty-cycle.



If a load step is applied on the second output (v_{02}), the controller will not detect this and therefore will the voltage decrease.

12) Compare the transformer utilization for a flyback, forward and a full-bridge. Comment on how the transformer is used in each converter, i.e. advantages/disadvantages. (4p)

	Flyback	Forward	Full-bridge
Transformer utilization	Unidirectional – only positive current fed into the transformer	Unidirectional – only positive current fed into the transformer	Bidirectional – uses switches to form an square-wave ac-voltage over the primary winding
Comment	The transformer acts as a mutual inductance that stores energy during the on-time. This requires an air-gap that lowers the magnetization inductance.	An extra winding is needed to demagnetize the transformer to the beginning of each period.	Bidirectional core utilization gives high power handling capability.

Formulas for Examination in Power Electronic Converters (ENM060)

Table 3-1 Use of Symmetry in Fourier Analysis

Symmetry	*Condition Required*	a_h and b_h
Even	$f(-t) = f(t)$	$b_h = 0 \quad a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\, d(\omega t)$
Odd	$f(-t) = -f(t)$	$a_h = 0 \quad b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\, d(\omega t)$
Half-wave	$f(t) = -f(t + \frac{1}{2}T)$	$a_h = b_h = 0$ for even h $a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\, d(\omega t)$ for odd h $b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\, d(\omega t)$ for odd h
Even quarter-wave	Even and half-wave	$b_h = 0$ for all h $a_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\cos(h\omega t)\, d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$
Odd quarter-wave	Odd and half-wave	$a_h = 0$ for all h $b_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\sin(h\omega t)\, d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$

Definition of RMS-value:

$$F_{RMS} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} f(t)^2 dt}$$

Definition of RMS-value with Fourier-series:

$$F_{RMS} = \sqrt{F_0^2 + \sum_{n=1}^{\infty} F_n^2} = \sqrt{\left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}}\right)^2}$$

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \qquad \sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \qquad \cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right) \qquad \sin(\alpha)\cos(\beta) = \frac{1}{2}\left(\sin(\alpha-\beta) + \sin(\alpha+\beta)\right)$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax), \ \int x\sin(ax)dx = \frac{1}{a^2}\left(\sin(ax) - ax\cos(ax)\right), \ \int \cos(ax)dx = \frac{1}{a}\sin(ax)$$

$$\int x\cos(ax)dx = \frac{1}{a^2}\left(\cos(ax) + ax\sin(ax)\right)$$

$$PF = \frac{P}{S} = \frac{V_s I_{s1}\cos\phi_1}{V_s I_s}, \ DPF = \cos\phi_1, \ \%THD_i = 100\frac{I_{dis}}{I_{s1}} = 100\frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = 100\sqrt{\sum_{h\neq 1}\left(\frac{I_{sh}}{I_{s1}}\right)^2}$$

Electromagnetics

$$e = \frac{d}{dt}\psi \qquad \psi = N\phi \qquad \phi = BA \qquad R = \frac{l}{A\mu_r\mu_0} \qquad L = \frac{\Psi}{i}$$

$$NI = R\phi = mmf \qquad N\phi = LI \qquad L = A_L N^2 \qquad W = \frac{1}{2}LI^2$$

Simpson's rule

Let $f(x)$ be a polynomial of maximum third degree, this means

$$f(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

For this function the integral can be calculated as

$$\frac{1}{T}\int_{t_0}^{t_0+T} f(x)dx = \frac{1}{6}\left(f(t_0) + 4f(t_0 + \frac{T}{2}) + f(t_0 + T)\right)$$