Midterm Exam **ENM060 Power Electronic Converters - Solutions**
Wednesday November 26, 2014

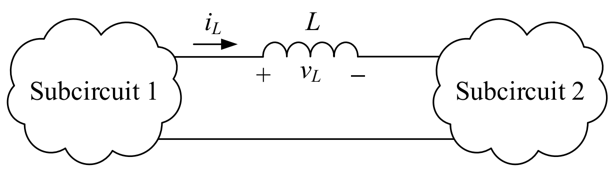
Lecturer: Andreas Karvonen, tel. 031-7721642 or 0709-524924
Help: CTH approved calculator (Casio FX82, Texas TI30, Sharp EL531)
Solutions: Will be posted on the course webpage (2014-11-26).
Mark list: Handed out 2014-12-03 at 15:15 in ML11
Pick-up of Exam Handed out 2014-12-03 at 15:15 in ML11

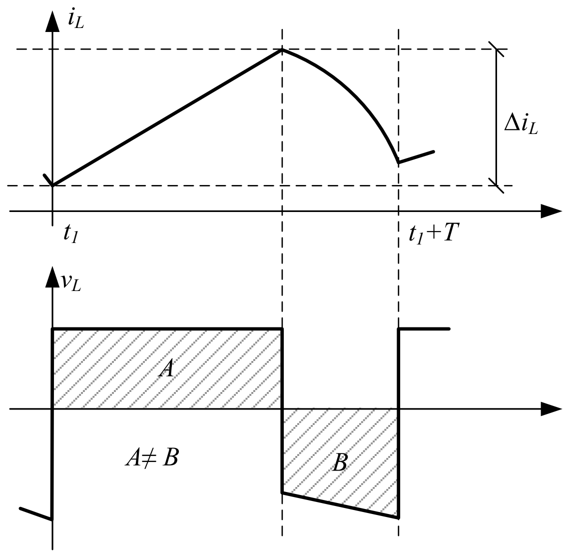
Each question is connected to a lecture (1 to 8). The bonus points are rewarded as follows:

-2p: 0-4p
+1p: 5-14p
+2p: 15-19p
+3p: 20-25p

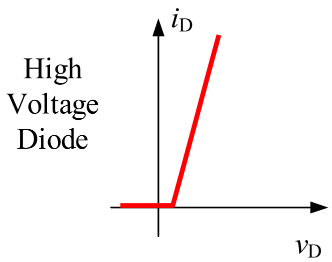
---

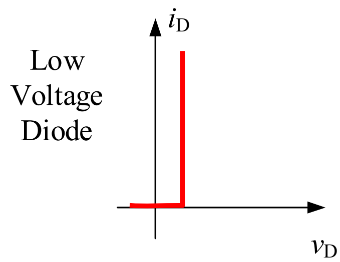
1. For the system below, draw the resulting inductor voltage and comment if the system is operating in steady-state. (3p)



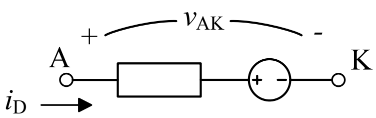


2. Draw the ideal *iv*-characteristics (i_D as a function of v_D) for a low voltage and a high voltage diode. Also, draw the equivalent circuits when the diodes are conducting. (4p)

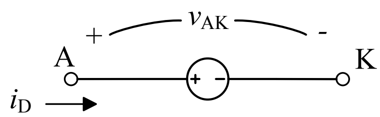




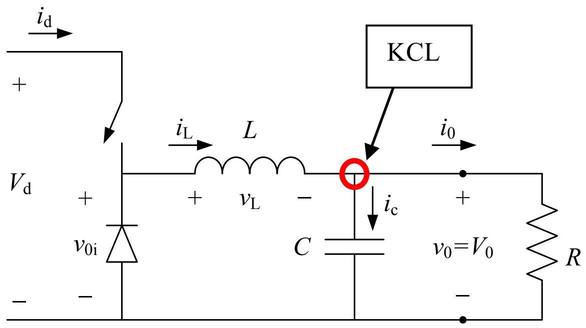
Equivalent circuit when conducting

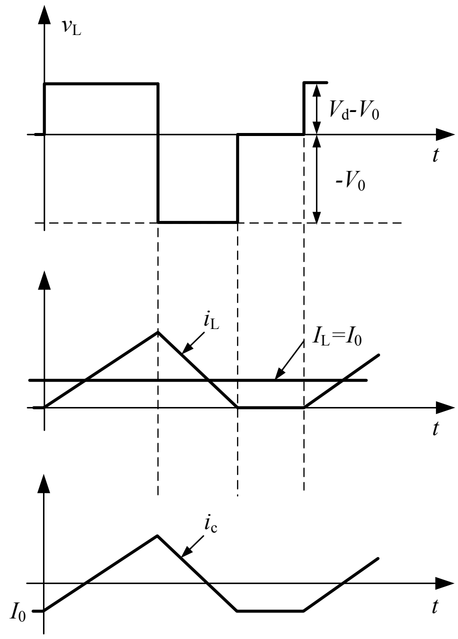


Equivalent circuit when conducting



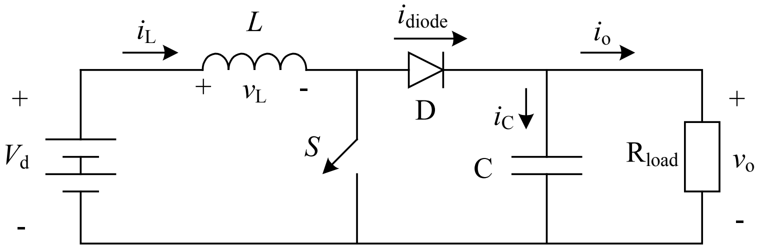
3. Apply kirchoffs current law on the marked node and draw the resulting currents (i_0, i_c and i_L). Assume that the output voltage is constant and that the converter is operating in discontinuous conduction mode (DCM). (3p)





If the converter is operating in steady state and in DCM, the current trough the inductor will become zero for a part of the total switching period. The inductor current then divides between the capacitor and the load where all the ripple current flows into the capacitor due to the fact that the output voltage is constant.

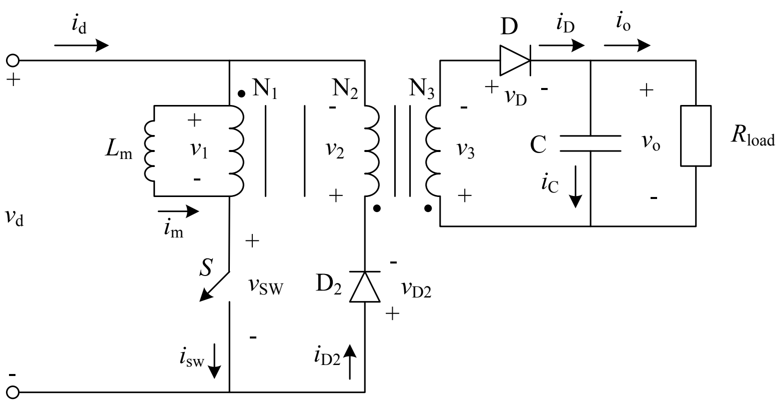
4. For a boost converter, derive an expression of the ratio between the input and output voltage when it is operating in continuous conduction mode (CCM). (3p)



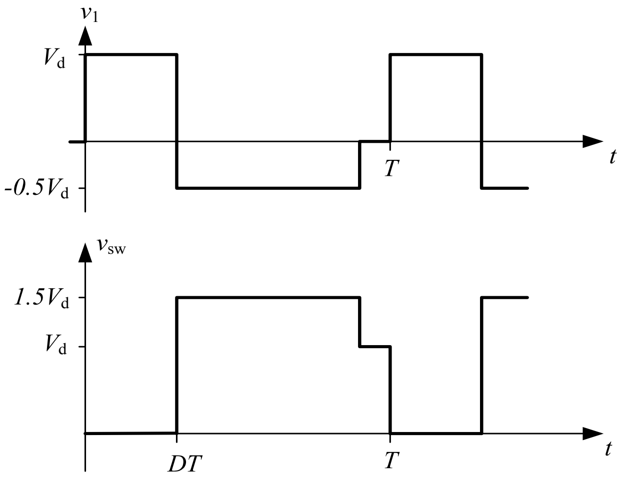
$$V_L = \frac{1}{T_{sw}}\int_0^{T_{sw}} v_L dt = \frac{1}{T_{sw}}(V_d D T_{sw} + (V_d - V_o)(1-D)T_{sw}) = V_d + V_o(1-D) = 0$$

$$V_o = \frac{V_d}{1-D} \quad \rightarrow \quad D = \frac{V_o - V_d}{V_o}$$

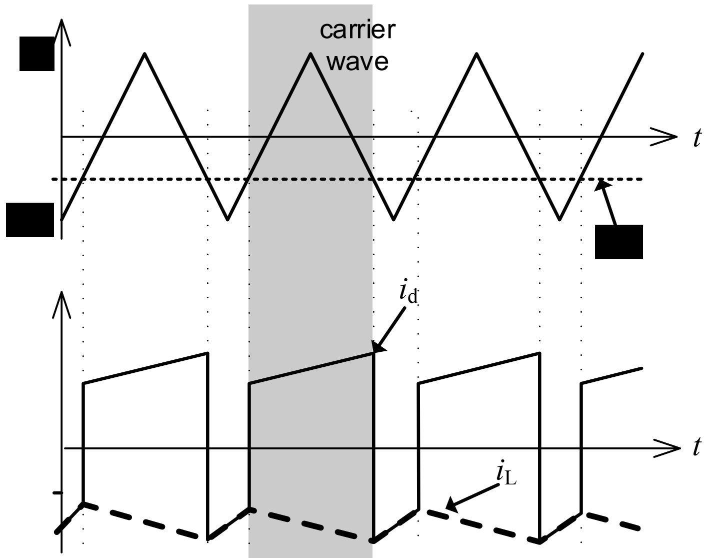
5. The flyback converter below has a protective winding (N_2) and the total turns ratio of the transformer (N_1: N_2: N_3) is (1: 2: 1). Draw the resulting switch voltage (v_{sw}) with D=0.3 if the converter is operating without any load (R_{load}=∞). (4p)



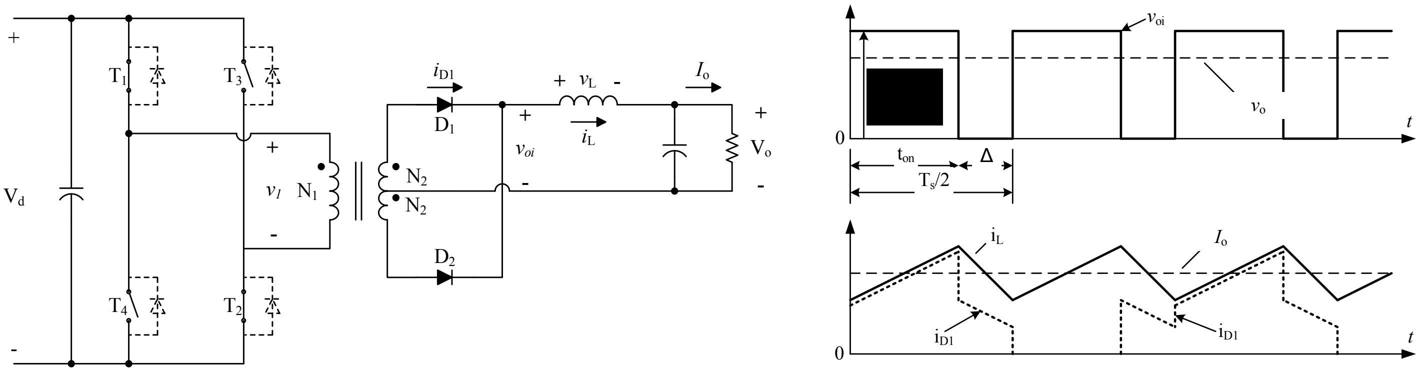
Due to the turns ratio of 1:2:1, the output voltage is limited to 0.5 times the input voltage. The duty-cycle is therefore limited to 0.3 and the resulting peak voltage over the switch will be $1.5V_d$.



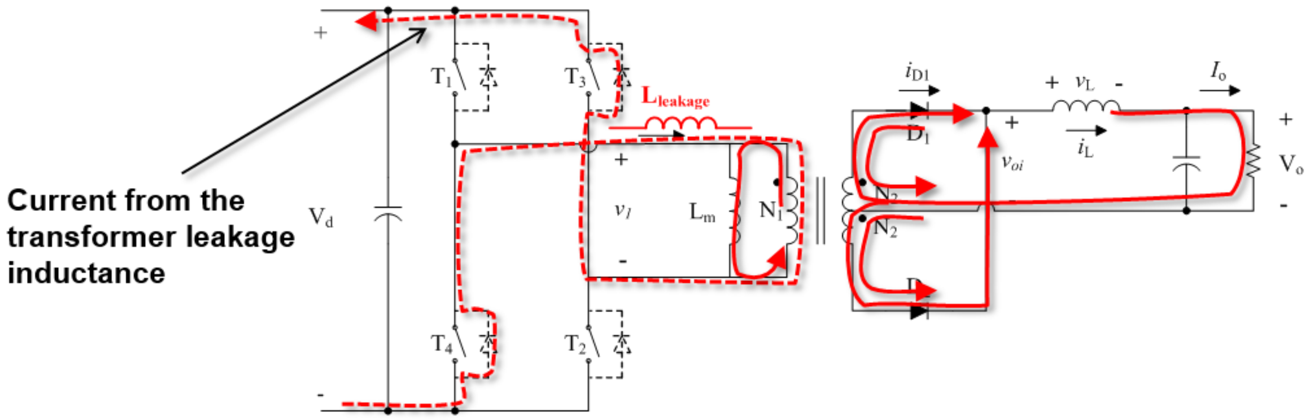
6. **The H-bridge converter below is operating with a bipolar switching pattern an output current that is negative during the entire switching period. Draw the resulting input (i_d) and output (i_o) current and draw the current path during the shaded time interval. (3p)**

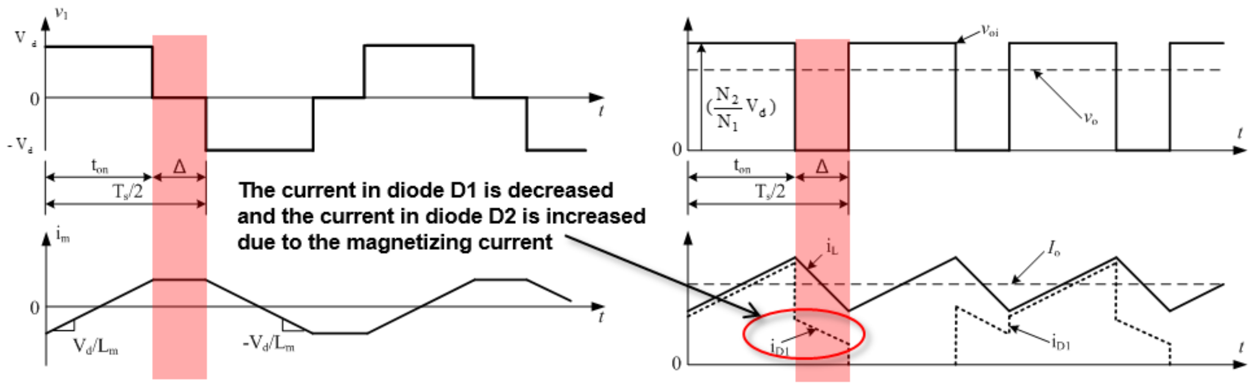


7. The fullbridge DC/DC converter below is realized with a real (non-ideal) transformer. Why must there be anti-parallel diodes connected over each switch? Explain and exemplify with e.g. a current arrow. (2p)

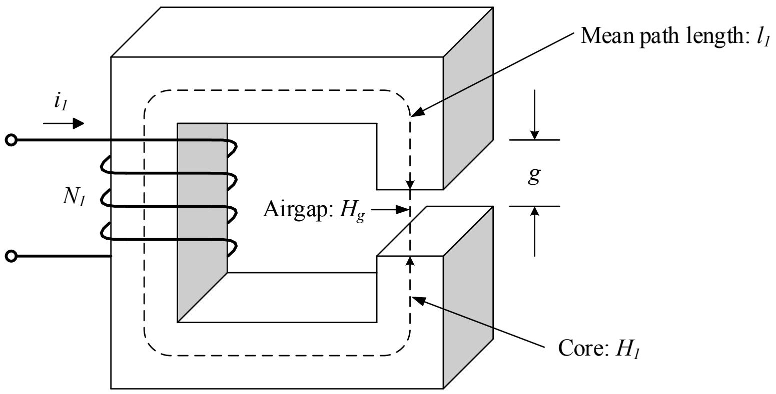


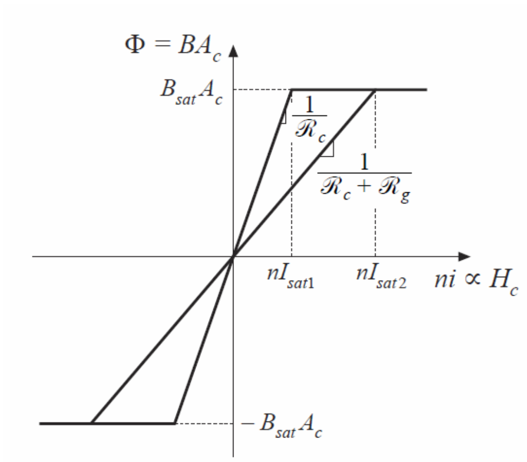
The anti parallel diodes must be implemented to handle the current from the transformer leakage inductance.





8. On the magnetic core depicted below, a coil is wound on one side. If an air gap is introduced in the core, how is the *BH*-loop affected? Explain with a suitable equation/expression and visualize in a *BH*-plot. (3p)





$$\left.\begin{aligned} ni &= \Phi\left(\Re_g + \Re_c\right)\\ L &= \frac{n^2}{\Re_g + \Re_c}\\ \Phi_{sat} &= B_{sat}A_c \end{aligned}\right\} I_{sat} = \frac{B_{sat}A_c}{n}\left(\Re_g + \Re_c\right)$$

Formulas for Examination in Power Electronic Converters (ENM060)

Table 3-1 Use of Symmetry in Fourier Analysis

Symmetry	*Condition Required*	a_h and b_h
Even	$f(-t) = f(t)$	$b_h = 0 \quad a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\, d(\omega t)$
Odd	$f(-t) = -f(t)$	$a_h = 0 \quad b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\, d(\omega t)$
Half-wave	$f(t) = -f(t + \frac{1}{2}T)$	$a_h = b_h = 0$ for even h $a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\, d(\omega t)$ for odd h $b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\, d(\omega t)$ for odd h
Even quarter-wave	Even and half-wave	$b_h = 0$ for all h $a_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\cos(h\omega t)\, d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$
Odd quarter-wave	Odd and half-wave	$a_h = 0$ for all h $b_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\sin(h\omega t)\, d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$

Definition of RMS-value:

$$F_{RMS} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} f(t)^2 dt}$$

Definition of RMS-value with Fourier-series:

$$F_{RMS} = \sqrt{F_0^2 + \sum_{n=1}^{\infty} F_n^2} = \sqrt{\left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2}}\right)^2}$$

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \qquad \sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \qquad \cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right) \qquad \sin(\alpha)\cos(\beta) = \frac{1}{2}\left(\sin(\alpha-\beta) + \sin(\alpha+\beta)\right)$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax), \quad \int x\sin(ax)dx = \frac{1}{a^2}\left(\sin(ax) - ax\cos(ax)\right), \quad \int \cos(ax)dx = \frac{1}{a}\sin(ax)$$

$$\int x\cos(ax)dx = \frac{1}{a^2}\left(\cos(ax) + ax\sin(ax)\right)$$

$$PF = \frac{P}{S} = \frac{V_s I_{s1}\cos\phi_1}{V_s I_s}, \quad DPF = \cos\phi_1, \quad \%THD_i = 100\frac{I_{dis}}{I_{s1}} = 100\frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = 100\sqrt{\sum_{h\neq 1}\left(\frac{I_{sh}}{I_{s1}}\right)^2}$$

Electromagnetics

$$e = \frac{d}{dt}\psi \qquad \psi = N\phi \qquad \phi = BA \qquad R = \frac{l}{A\mu_r\mu_0} \qquad L = \frac{\Psi}{i}$$

$$NI = R\phi = mmf \qquad N\phi = LI \qquad L = A_L N^2 \qquad W = \frac{1}{2}LI^2$$

Simpson's rule

Let $f(x)$ be a polynomial of maximum third degree, this means

$$f(x) = a_1 + a_2 x + a_3 x^2 + a_4 x^3$$

For this function the integral can be calculated as

$$\frac{1}{T}\int_{t_0}^{t_0+T} f(x)dx = \frac{1}{6}\left(f(t_0) + 4f(t_0 + \frac{T}{2}) + f(t_0 + T)\right)$$