Svar till tentamen 010523

1)

$$\left|\sin(2\pi t)\right| = \frac{2}{\pi} - \frac{4}{\pi}\sum_n \frac{\cos(2n\cdot 2\pi t)}{4n^2-1}$$

Lågpass: 4πn<30

$$\Rightarrow y(t) = \frac{2}{\pi} - \frac{4}{3\pi}\cos(4\pi t) - \frac{4}{15\pi}\cos(8\pi t)$$

Medelvärde, samma som likspänningsvärde:

$x_m = 0$; $y_m = 2/\pi$

Effektivvärde:

$$x_{eff} = \frac{1}{\sqrt{2}} \approx 0.707$$

Kvadratisk addition av komponenternas effektivvärden:

$$y_{eff} = \frac{1}{\pi}\sqrt{4+\frac{8}{9}+\frac{8}{225}} \approx 0.706$$

2)

$$H(s) = k\frac{(s+j)(s-j)}{(s+2)(s+1+j)(s+1-j)} = k\frac{s^2+1}{s^3+4s^2+6s+4}$$

$$H(0) = 1 = k\frac{1}{4} \Rightarrow k = 4$$

$$H(s) = \frac{4(s^2+1)}{s^3+4s^2+6s+4} = \frac{A}{s+2} + \frac{B}{s+1+j} + \frac{C}{s+1-j}$$

$$A = \left.\frac{4(s^2+1)}{s^2+2s+s}\right|_{s=-2} = \ldots = 10$$

$$B = \left.\frac{4(s^2+1)}{(s+2)(s+1-j)}\right|_{s=-1-j} = \ldots = 3+j = \sqrt{10}e^{-j2.82}$$

$$C = B^* = \sqrt{10}e^{j2.82}$$

$$H(s) = \frac{10}{s+2} + \frac{\sqrt{10}e^{-j2.82}}{s+1+j} + \frac{\sqrt{10}e^{j2.82}}{s+1-j}$$

$$h(t) = 10e^{-2t}u(t) + 2\sqrt{10}e^{-t}\cos(t+2.82)u(t)$$

$$H(\omega) = \frac{4(1-\omega^2)}{4(1-\omega^2)+j\omega(6-\omega^2)}$$

3)

a) FFT speglad runt halva längden: 2 sinusar

$$f = f_s \frac{k}{N} \Rightarrow f_1 = 10^4 \frac{51}{512} = 996 \approx 1.0 \text{ kH}$$

$$f_2 = 10^4 \frac{128}{512} = 2.5 \text{ kH}$$

b) $\Delta f = \frac{f_s}{N}$ För att kunna se två närliggande frekvenser (f_a, f_b) måste frekvensupplösningen vara minst $\frac{|f_a - f_b|}{2}$

$$\frac{10}{2} > \frac{f_s}{N} \quad \Rightarrow \quad N > \frac{f_s}{5} = 2000$$

$$FFT \Rightarrow N = 2^p \Rightarrow N = 2^{11} = 2048 > \frac{f_s}{5}$$

4)

a) y[n] - 1.3y[n-1] + 0.4y[n-2] = x[n] + x[n-1]

b) $H(z) = \frac{z(z+1)}{z^2 - 1.3z + 0.4} = \frac{z(z+1)}{(z-0.8)(z-0.5)}$

c) Polerna innanför enhetscirkeln: Stabilt system

d)

$$\frac{H(z)}{z} = \frac{z+1}{(z-0.8)(z-0.5)} = \frac{A}{z-0.8} + \frac{B}{z-0.5}$$

$$A = \left.\frac{z+1}{z-0.5}\right|_{z=0.8} = \frac{1.8}{0.3} = 6$$

$$B = \left.\frac{z+1}{z-0.8}\right|_{z=0.5} = \frac{1.5}{-0.3} = -5$$

$$H(z) = \frac{6z}{z-0.8} - \frac{5z}{z-0.5}$$

$$h[n] = 6(0.8)^n u[n] - 5(0.5)^n u[n]$$

e)

f)

5)

Butterworth, 3:e ordn:

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$\omega_c = 2\pi f_{3dB} = 200\pi$$

högpass: $s \to \frac{\omega_c}{s}$

$$H(s) = \frac{1}{\left(\frac{\omega_c}{s}\right)^3 + 2\left(\frac{\omega_c}{s}\right)^2 + 2\frac{\omega_c}{s} + 1} = \frac{s^3}{s^3 + 2\omega_c s^2 + 2\omega_c{}^2 s + \omega_c{}^3}$$