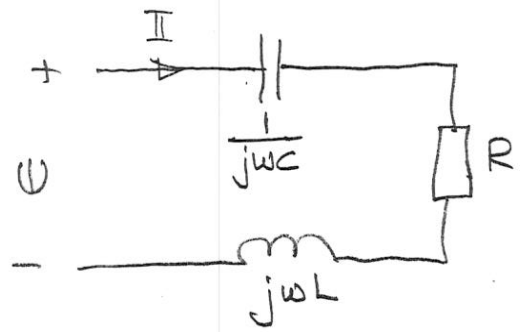
Lösningar Kretsanalys aug 2013

1.



$i(t) = 100\cos(2000t - 10^\circ)$ mA

$\Rightarrow \mathbb{I} = 0{,}1\angle{-10^\circ}$ A

$R = 330\,\Omega$

$L = 150$ mH, $C = 820$ nF

• $\mathbb{U} = \mathbb{Z}\cdot\mathbb{I}$, $\mathbb{Z} = R + j\omega L + \frac{1}{j\omega C} = (\omega = 2000 \text{ rad/s}) =$

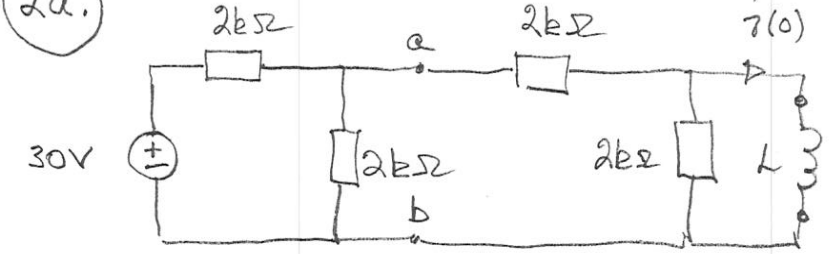
$= 330 + j\,300 - j\,609{,}8 = 330 - j\,309{,}8 = 452{,}6\angle{-43{,}2^\circ}\,\Omega$

$\Rightarrow \mathbb{U} = 452{,}6\angle{-43{,}2^\circ}\,\Omega \cdot 0{,}1\angle{-10^\circ}\text{ A} = 45{,}3\angle{-53{,}2^\circ}\text{ V} \Rightarrow$

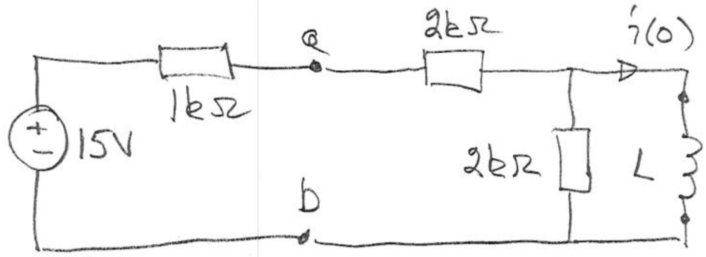
$\underline{\underline{u(t) = 45{,}3\cos(2000t - 53{,}2^\circ)\text{ V}}}$

• $P = R\cdot I_{rms}^2 = 330\,\Omega\cdot\left(\frac{0{,}1\text{A}}{\sqrt{2}}\right)^2 = \underline{1{,}65\text{ W}}$

2a.

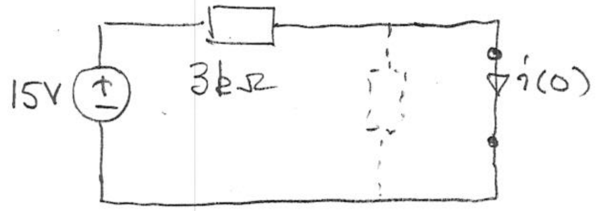


$\Leftrightarrow$ (Tvåpols-omvandling)



Stationärtillstånd DC $\Rightarrow$ Spole = kortslutn.

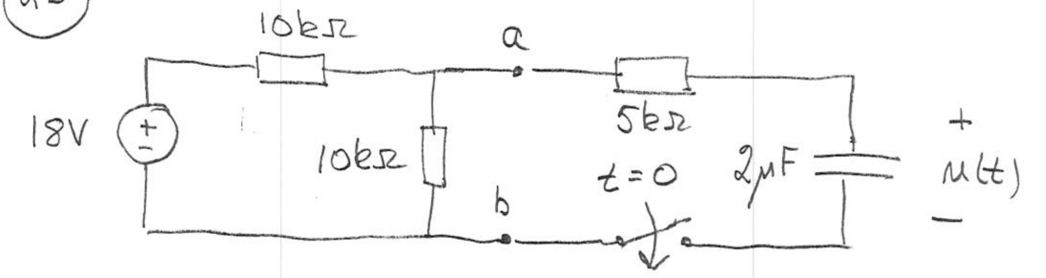
$\Leftrightarrow$



$\Rightarrow$

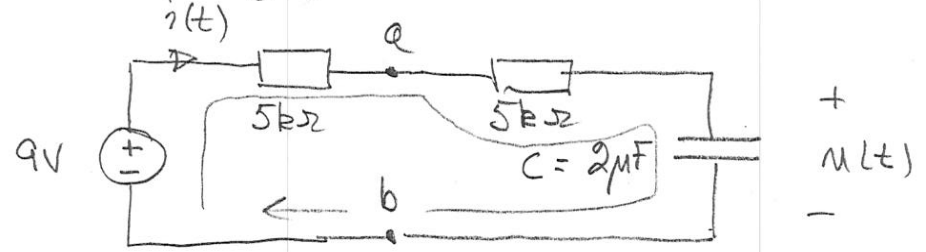
$$i(0) = \frac{15\,\text{V}}{3\,\text{k}\Omega} = \underline{\underline{5\,\text{mA}}}$$

(2b)



$\Leftrightarrow$

(Tvåpolsomvandling), Schema för $t > 0$.



, $u(0) = -5V$

• KVL ger $+9V - 5k\Omega \cdot i(t) - 5k\Omega \cdot i(t) - u(t) = 0$

• $i(t) = 2\mu F \cdot \frac{du}{dt} \Rightarrow$ (utan enheter)

$$9 - 10 \cdot 10^3 \cdot 2 \cdot 10^{-6} \frac{du}{dt} - u(t) = 0 \Rightarrow \frac{du}{dt} + 50u(t) = 450$$

$u(t) = u_h(t) + u_p(t)$

$u_h(t)$ $\quad \frac{du}{dt} + 50u(t) = 0 \Rightarrow u_h(t) = A \cdot e^{-50t}$

$u_p(t)$ Ansätt $u_p(t) = B \Rightarrow$

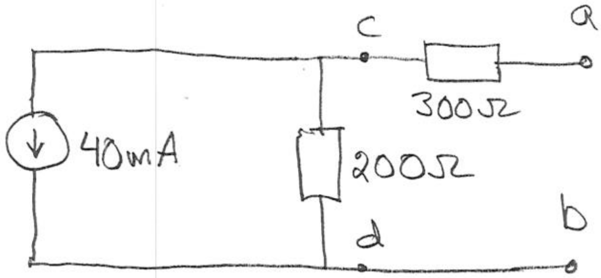
$0 + 50B = 450 \Rightarrow B = 9$, så att

$u(t) = 9 + A \cdot e^{-50t}$, $u(0) = -5V \Rightarrow$

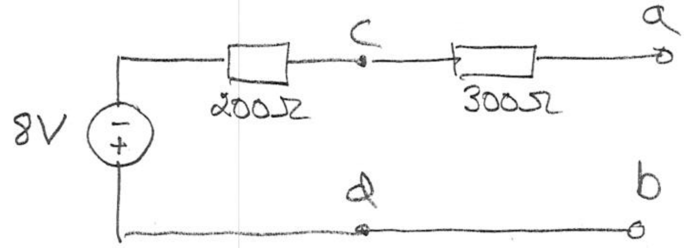
$-5 = 9 + A \cdot e^0 \Rightarrow A = -14 \Rightarrow$

$$\underline{\underline{u(t) = 9 - 14e^{-50t}\ V}}$$

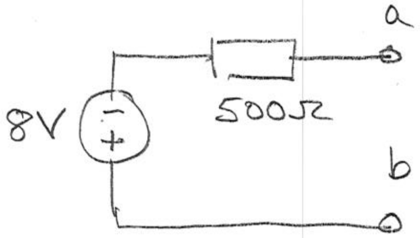
3a



$\Longleftrightarrow$



$\Longleftrightarrow$

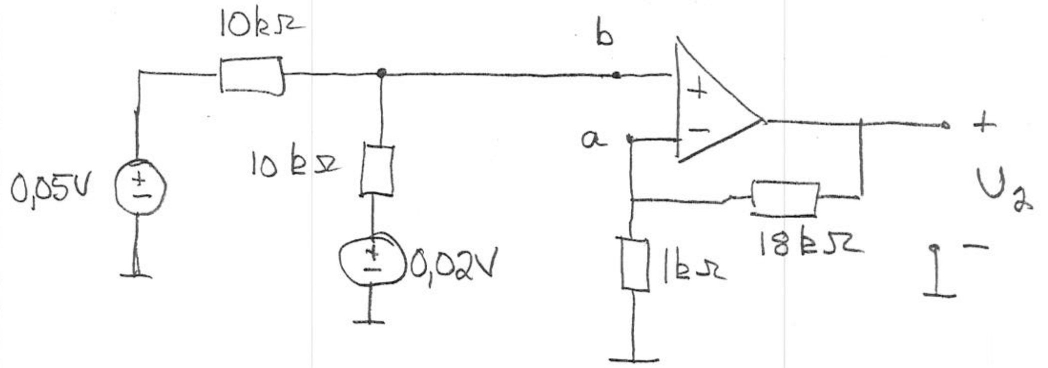


$\Longleftrightarrow$

Maximal effekt i en resistor på 500Ω ansluten mellan a och b:

$$P_{max} = \frac{\left(\frac{8V}{2}\right)^2}{500\Omega} = \underline{\underline{32mW}}.$$

(3b.)



Ideal motkopplad OP $\Longrightarrow$ · inströmmar = 0

· $V_a = V_b$

Nodanalys

a. $$\frac{V_a}{1k\Omega} + \frac{V_a - U_2}{18k\Omega} = 0 \Longrightarrow V_a\left(\frac{1}{1k\Omega} + \frac{1}{18k\Omega}\right) = \frac{U_2}{18k\Omega}$$

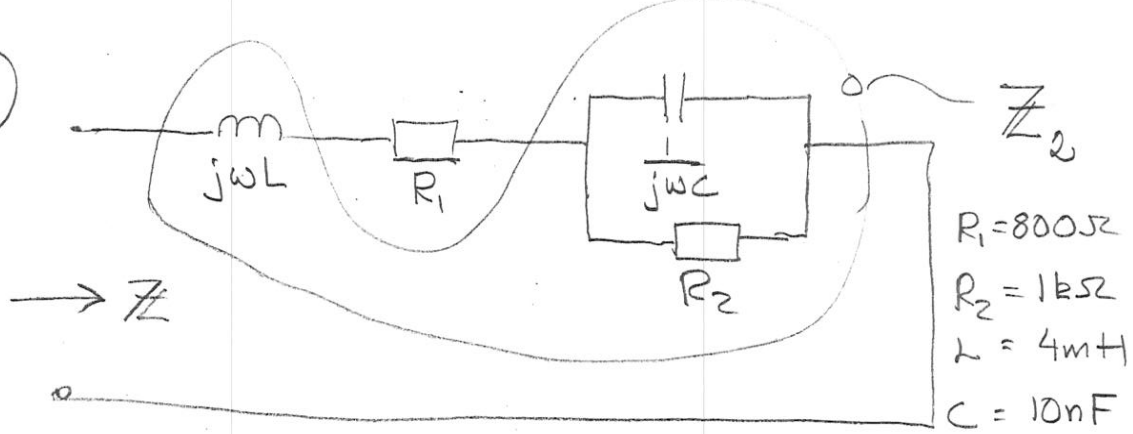
$$\Longrightarrow U_2 = (18+1) \cdot V_a = 19 V_a$$

b. $$\frac{V_b - 0{,}05V}{10k\Omega} + \frac{V_b - 0{,}02V}{10k\Omega} = 0 \Longrightarrow$$

$$2V_b = 0{,}05V + 0{,}02V \Longrightarrow V_b = 0{,}035V$$

$$V_a = V_b \Longrightarrow U_2 = 19 \cdot 0{,}035V = \underline{\underline{0{,}665V}}$$

4.



$$\bullet\ Z = R_1 + Z_2 \;;\; Z_2 = j\omega L + R_2 \,//\, \frac{1}{j\omega C} =$$

$$= j\omega L + \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = j\omega L + \frac{R_2}{1 + j\omega R_2 C} =$$

$$= \frac{j\omega L - \omega^2 R_2 L C + R_2}{1 + j\omega R_2 C} = \frac{R_2 - \omega^2 R_2 L C + j\omega L}{1 + j\omega R_2 C}$$

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a. Resonans $\Leftrightarrow$ Z reell $\Leftrightarrow$ Z_2 reell

$$\Rightarrow \frac{R_2 - \omega^2 R_2 L C}{1} = \frac{\omega L}{\omega R_2 C} \Rightarrow$$

$$R_2(1 - \omega^2 L C) = \frac{L}{R_2 C} \Rightarrow \omega^2 L C = 1 - \frac{L}{R_2^2 C} \Rightarrow$$

$$\omega = \frac{1}{\sqrt{LC}} \cdot \sqrt{1 - \frac{L}{R_2^2 C}} = \frac{1}{\sqrt{4 \cdot 10^{-3} \cdot 10 \cdot 10^{-9}}} \cdot \sqrt{1 - \frac{4 \cdot 10^{-3}}{1000^2 \cdot 10 \cdot 10^{-9}}} =$$

$$= 158{,}1 \cdot 10^{3} \cdot \sqrt{1 - 0{,}4}\ \text{rad/s} = \underline{\underline{122\ \text{krad/s}}}$$

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b. $Z_{resonans} = R_1 + Z_{2\,resonans} =$

$$= R_1 + \frac{L}{R_2 C} = 800\Omega + 400\Omega = \underline{\underline{1{,}2\,k\Omega}}$$

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c. Resonans saknas om

$$1 - \frac{L}{R_2^2 C} < 0 \iff \frac{L}{R_2^2 C} > 1 \iff$$

$$C < \frac{L}{R_2^2} = \frac{4 \cdot 10^{-3}}{1000^2} = \underline{\underline{4\,nF}}$$

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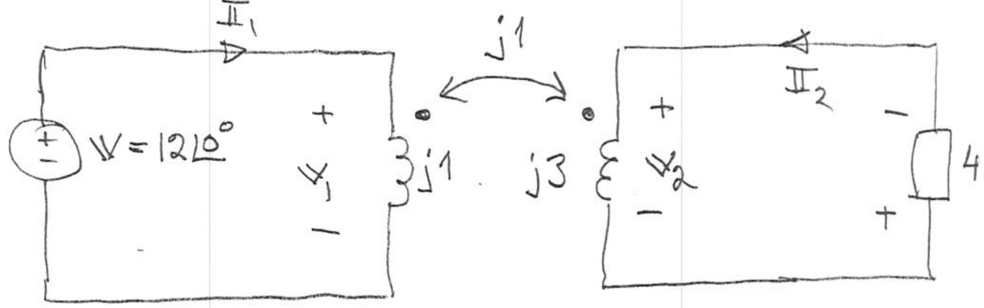
5. Komplext beräkningsschema



$$\left.\begin{array}{l} \bullet\ \mathbb{V}_1 = \mathbb{V} = 12\angle 0^\circ \\ \bullet\ \mathbb{V}_1 = j1\cdot\mathbb{I}_1 + j1\,\mathbb{I}_2 \end{array}\right\} \; j(\mathbb{I}_1 + \mathbb{I}_2) = 12\angle 0^\circ \Rightarrow$$

$$\mathbb{I}_1 + \mathbb{I}_2 = -j\cdot 12\angle 0^\circ = -j12 \quad (1)$$

$$\left.\begin{array}{l} \bullet\ \mathbb{V}_2 = -4\cdot\mathbb{I}_2 \\ \bullet\ \mathbb{V}_2 = j3\,\mathbb{I}_2 + j1\,\mathbb{I}_1 \end{array}\right\} \Rightarrow -4\mathbb{I}_2 = j3\,\mathbb{I}_2 + j\,\mathbb{I}_1 \Rightarrow$$

$$j\,\mathbb{I}_1 + (4+j3)\mathbb{I}_2 = 0 \quad (2). \quad (1) \text{ och } (2) \Rightarrow$$

$$j\,\mathbb{I}_1 + (4+j3)\cdot(-j12 - \mathbb{I}_1) = 0 \Rightarrow$$

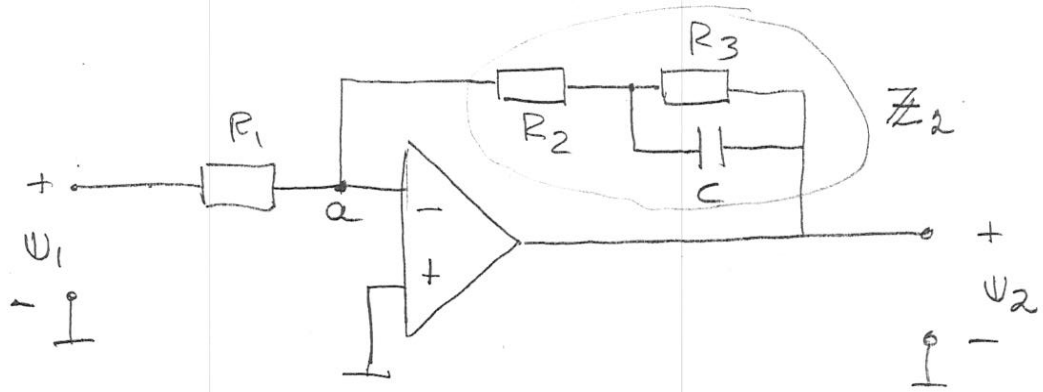
$$j\,\mathbb{I}_1 - j48 + 36 - 4\mathbb{I}_1 - j3\,\mathbb{I}_1 = 0 \Rightarrow$$

$$(-4 - j2)\mathbb{I}_1 = -36 + j48 \Leftrightarrow (4+j2)\mathbb{I}_1 = 36 - j48 \Rightarrow$$

$$\mathbb{I}_1 = \frac{36 - j48}{4+j2} = \frac{60\angle -53{,}1^\circ}{4{,}47\angle 26{,}6^\circ} = 13{,}4\angle -79{,}7^\circ \Rightarrow$$

$$i_1(t) = 13{,}4\cos(1000t - 79{,}7^\circ)\text{ A}$$

6.



$R_1 = 2{,}2\,k\Omega$, $R_2 = 18\,k\Omega$, $R_3 = 390\,k\Omega$, $C = 4{,}7\,nF$

Ideal motkopplad OP ⇒ • inströmmar = 0
• $V_a = 0$

$$\bullet\; Z_2 = R_2 + R_3 /\!/ \frac{1}{j\omega C} = R_2 + \frac{R_3 \cdot \frac{1}{j\omega C}}{R_3 + \frac{1}{j\omega C}} =$$

$$= R_2 + \frac{R_3}{1 + j\omega R_3 C} = \frac{R_2 + R_3 + j\omega R_2 R_3 C}{1 + j\omega R_3 C} =$$

$$= (R_2 + R_3) \cdot \frac{1 + j\omega \frac{R_2 R_3}{R_2 + R_3} C}{1 + j\omega R_3 C}$$

Nodanalys i a:

$$\frac{V_a - U_1}{R_1} + \frac{V_a - U_2}{Z_2} = 0 \;;\; V_a = 0 \implies$$

$$U_2 = -\frac{Z_2}{R_1} \cdot U_1 \implies \frac{U_2}{U_1} = -\frac{Z_2}{R_1} =$$

$$= -\frac{R_2 + R_3}{R_1} \cdot \frac{1 + j\omega \frac{R_2 R_3}{R_2 + R_3} C}{1 + j\omega R_3 C} = -185{,}9 \cdot \frac{1 + j\frac{\omega}{12{,}4 \cdot 10^3}}{1 + j\frac{\omega}{546}}$$

$\implies \underline{\underline{\omega_1 = 546\,rad/s}}$; $\underline{\underline{\omega_2 = 12{,}4\,krad/s}}$

$\omega << 546 \Rightarrow$

$$\frac{U_2}{U_1} \approx -185{,}9 \cdot \frac{1+j0}{1+j0} = -185{,}9 \Rightarrow$$

$$A = 20 \cdot \log|-185{,}9| = \underline{\underline{45{,}4\,dB}}$$

$\omega >> 12{,}4$ krad/s $\Rightarrow$

$$\frac{U_2}{U_1} \approx -185{,}9 \cdot \frac{\frac{\omega}{12{,}4 \cdot 10^3}}{\frac{\omega}{546}} = -8{,}19 \Rightarrow$$

$$B = 20 \cdot \log|-8{,}19| = \underline{\underline{18{,}3\,dB}}$$