# Lösningar Kretsanalys april 2013

**1a.** $\underline{V} = 325\angle 0^\circ\ A \qquad S = 1500 + j\,3000\,VA = 3354\angle 63{,}4^\circ\ VA$

$$S = \frac{\underline{V}\cdot\underline{I}^*}{2} \Rightarrow \underline{I}^* = \frac{2S}{\underline{V}} = \frac{2\cdot 3354\angle 63{,}4^\circ}{325\angle 0^\circ}\ A = 20{,}64\angle 63{,}4^\circ\ A$$

$$\Rightarrow \underline{I} = 20{,}64\angle -63{,}4^\circ\ A \Rightarrow$$

$$i(t) = 20{,}64\cos(314t - 63{,}4^\circ)\ A$$

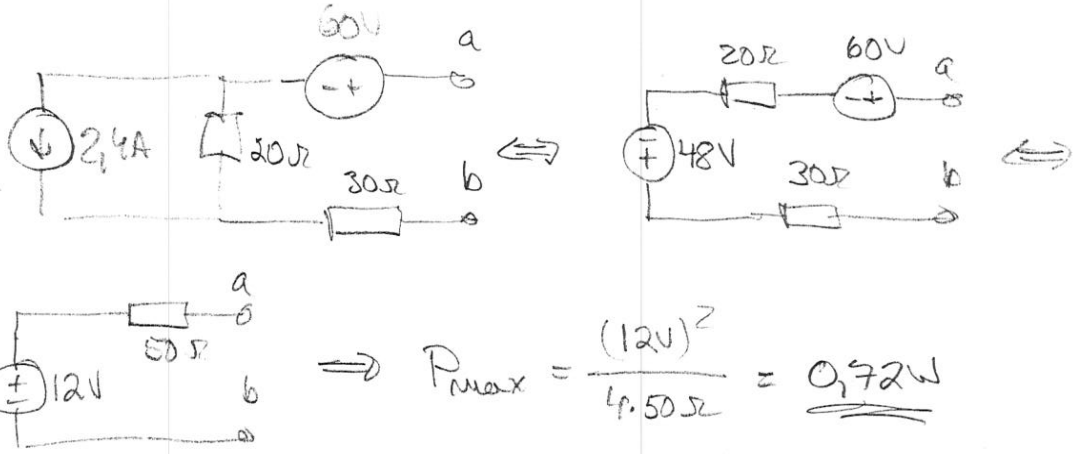
**1b.**

$$\underline{Z} = \frac{\underline{V}}{\underline{I}} = \frac{325\angle 0^\circ\ V}{20{,}64\angle -63{,}4^\circ\ A} = 15{,}7\angle -63{,}4^\circ\ \Omega =$$

$= 7{,}05 + j14{,}1\ \Omega = R + j\omega L$ eftersom kretsen är induktiv $\Rightarrow$

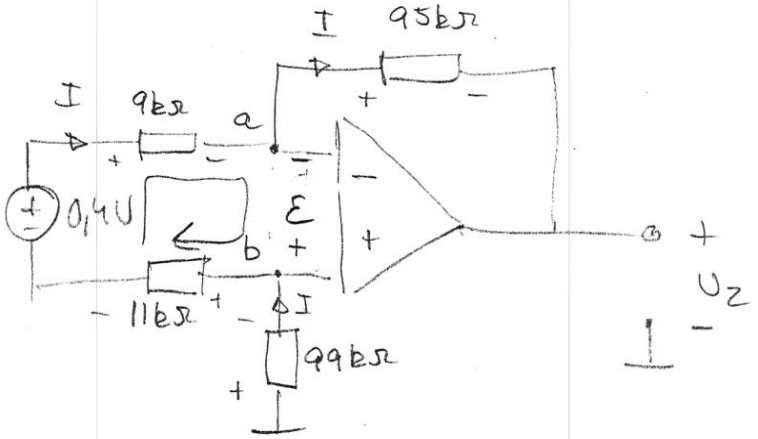
$$R = 7{,}05\ \Omega \qquad L = \frac{X}{\omega} = \frac{14{,}1}{314}\ H = 44{,}8\ mH$$

3a




$$\Rightarrow P_{max} = \frac{(12V)^2}{4 \cdot 50\Omega} = \underline{\underline{0,72W}}$$

3b




Ideal rückgekoppelt OP $\Rightarrow$ · Ströme $= 0$

· $V_a = V_b = 0 \Leftrightarrow \varepsilon = 0$.

· $U_2 = V_a - 95k\Omega \cdot I$

· $V_b = V_b$

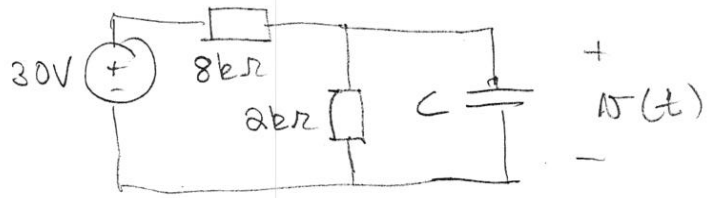
· I gen. auf Kirchhoff: $+0,4V - 9k\Omega \cdot I + \varepsilon - 11k\Omega \cdot I = 0$

$$\Rightarrow I = \frac{0,4V}{9k\Omega + 11k\Omega} = 20\mu A$$

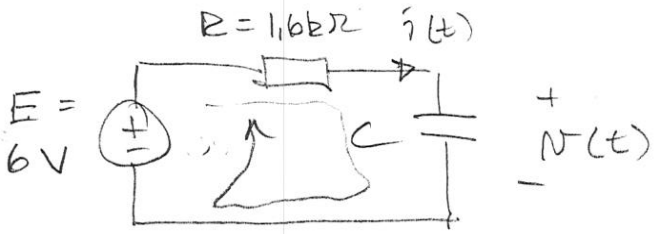
· $V_b = 0 - 99k\Omega \cdot 20\mu A = -1,98V \Rightarrow$

$$U_2 = -1,98V - 95k\Omega \cdot 20\mu A = \underline{\underline{-3,88V}}$$

2. Schema for $t > 0$




$\Leftrightarrow$




$C = 1\mu F$

$v(t)$ ges av Kirchhoff:

$+E - R \cdot i(t) - v(t) = 0$, där $i(t) = C \cdot \frac{dv}{dt} \Rightarrow$

$E - RC\frac{dv}{dt} - v(t) = 0 \Rightarrow \frac{dv}{dt} + \frac{1}{RC} \cdot v(t) = \frac{E}{RC}$

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$\underline{v_h(t)}$ $\quad \frac{dv}{dt} + \frac{1}{RC} \cdot v = 0 \Rightarrow v = A \cdot e^{-\frac{t}{RC}}$

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$\underline{v_p(t)}$ Ansätt $v_p(t) = B$ (konstant) $\Rightarrow$

$0 + \frac{1}{RC} \cdot B = \frac{E}{RC} \Rightarrow B = E \Rightarrow$

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$v(t) = v_h(t) + v_p(t) = A \cdot e^{-\frac{t}{RC}} + E$

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$v(0) = 30V \Rightarrow$

$30V = A \cdot 1 + E = A + 6V \Rightarrow A = 24V \Rightarrow$

$v(t) = 24 \cdot e^{-\frac{t}{RC}} + 6V = \underline{24 \cdot e^{-\frac{t}{1,6ms}} + 6V}$

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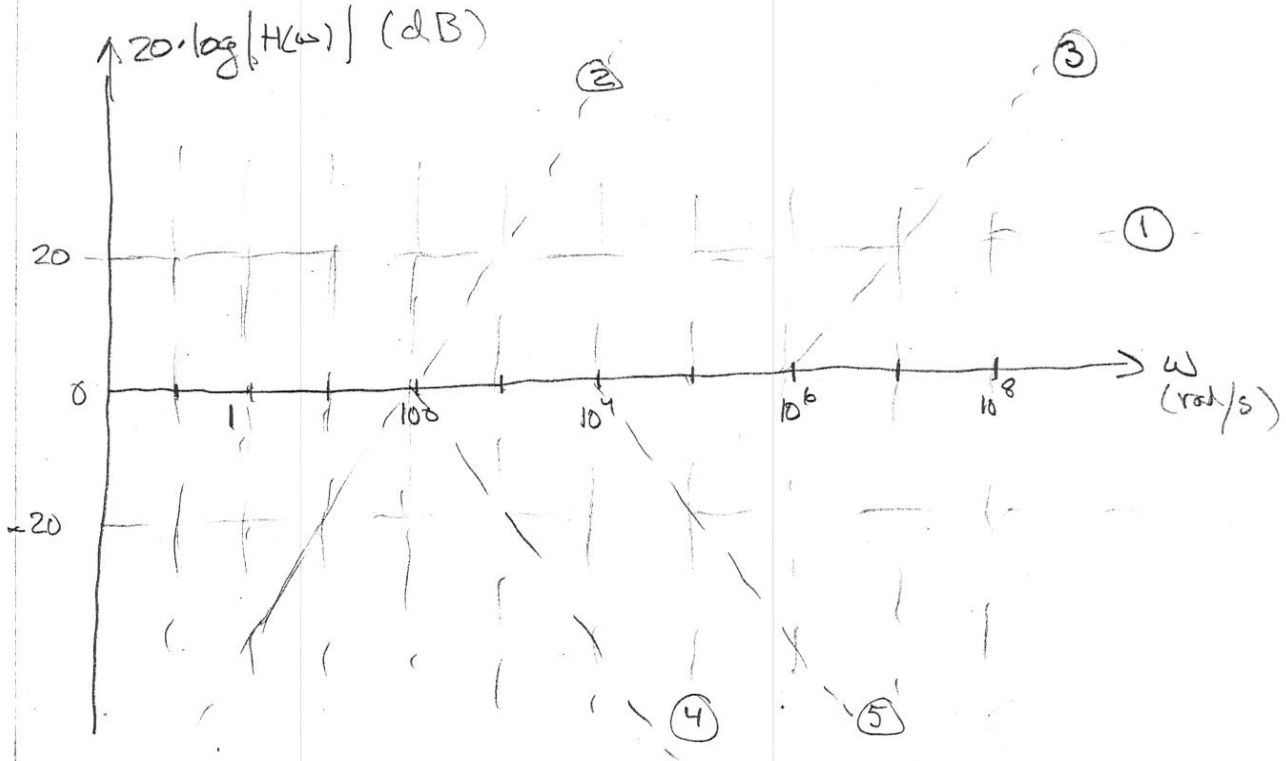
4a

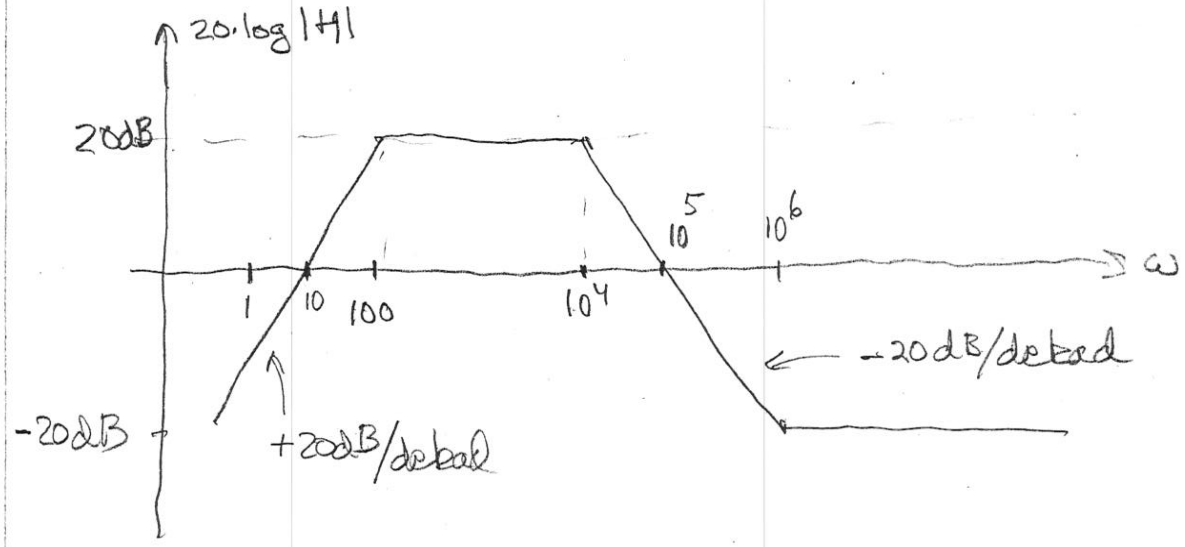
$$H(\omega) = \frac{j\omega(10^6 + j\omega)}{(1000 + j10\omega)(10^4 + j\omega)} = \frac{j\omega \cdot 10^6\left(1 + j\frac{\omega}{10^6}\right)}{1000\left(1 + j\frac{10\omega}{1000}\right)\cdot 10^4\left(1 + j\frac{\omega}{10^4}\right)} =$$

$$= \frac{1}{10} \cdot \frac{j\omega\left(1 + j\frac{\omega}{10^6}\right)}{\left(1 + j\frac{\omega}{100}\right)\left(1 + j\frac{\omega}{10^4}\right)} = 10 \cdot \frac{\overset{(3)}{j\frac{\omega}{100}}\left(1 + j\frac{\omega}{10^6}\right)^{(3)}}{\left(1 + j\frac{\omega}{100}\right)\left(1 + j\frac{\omega}{10^4}\right)}$$

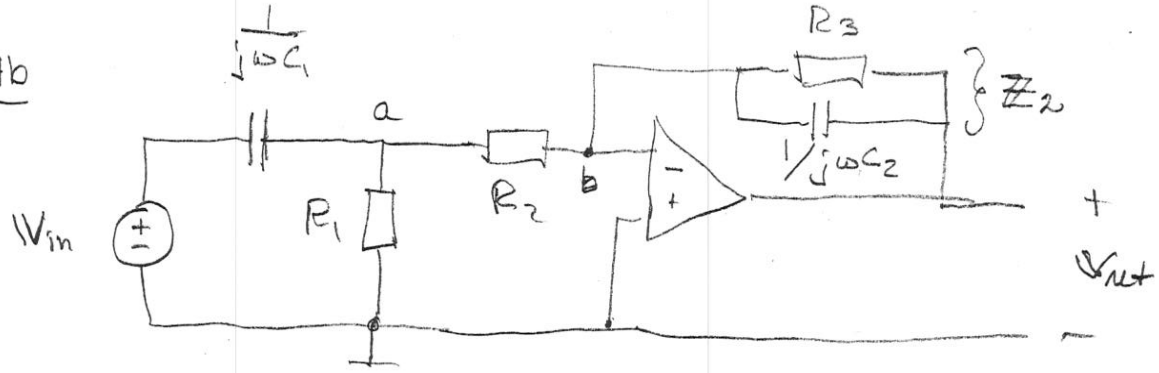
(1) (4) (5)




$\Rightarrow$



4b




Ideal motkopplad OP ⇒ · inströmmar = 0

· $V_b = 0$

Nodanalys:

a. $$\frac{V_a - V_{in}}{1/j\omega C_1} + \frac{V_a}{R_1} + \frac{V_a - V_b}{R_2} = 0 \;;\; V_b = 0 \Rightarrow$$

$$V_a\left(j\omega C_1 + \frac{1}{R_1} + \frac{1}{R_2}\right) = j\omega C_1 V_{in} \Rightarrow$$

$$V_a = \frac{j\omega R_1 R_2 C_1}{j\omega R_1 R_2 C_1 + R_1 + R_2} V_{in} = \frac{j\omega \frac{R_1 R_2}{R_1+R_2} C_1}{1 + j\omega \frac{R_1 R_2}{R_1+R_2} C_1} \quad (1)$$

b. $$\frac{V_b - V_a}{R_2} + \frac{V_b - V_{ut}}{Z_2} = 0 \;;\; V_b = 0 \Rightarrow$$

$$V_{ut} = -\frac{Z_2}{R_2} \cdot V_a \;;\; Z_2 = R_3 // \frac{1}{j\omega C_2} = \frac{R_3 \cdot \frac{1}{j\omega C_2}}{R_3 + \frac{1}{j\omega C_2}} =$$

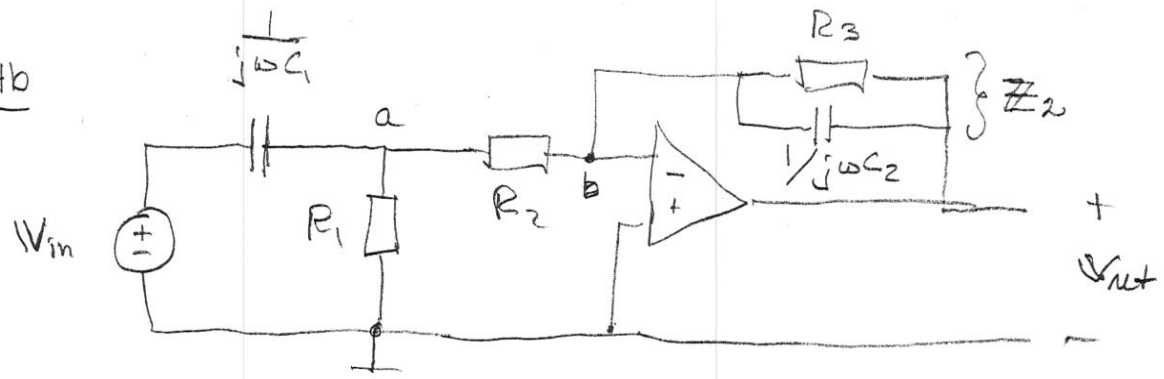
$$= \frac{R_3}{1 + j\omega R_3 C_2} \cdot (1) \Rightarrow$$

$$V_{ut} = -\frac{R_3}{R_2} \cdot \frac{1}{1 + j\omega R_3 C_2} \cdot V_a = -\frac{R_3}{R_2} \cdot \frac{j\omega \frac{R_1 R_2}{R_1+R_2} C_1}{(1 + j\omega R_3 C_2)\left(1 + j\omega \frac{R_1 R_2}{R_1+R_2} C_1\right)} =$$

$$= -10 \cdot \frac{j\frac{\omega}{500}}{\left(1 + j\frac{\omega}{2\cdot 10^5}\right)\left(1 + j\frac{\omega}{500}\right)} \Rightarrow$$

$K = -10$ $\quad \omega_1 = 500$ rad/s $\quad \omega_2 = 200\,000$ rad/s

4b




Ideal motkopplad OP ⇒ · inströmmar = 0

· $V_b = 0$

Nodanalys:

a. $$\frac{V_a - V_{in}}{1/j\omega C_1} + \frac{V_a}{R_1} + \frac{V_a - V_b}{R_2} = 0 \;;\; V_b = 0 \Rightarrow$$

$$V_a\left(j\omega C_1 + \frac{1}{R_1} + \frac{1}{R_2}\right) = j\omega C_1 V_{in} \Rightarrow$$

$$V_a = \frac{j\omega R_1 R_2 C_1}{j\omega R_1 R_2 C_1 + R_1 + R_2} V_{in} = \frac{j\omega \frac{R_1 R_2}{R_1+R_2} C_1}{1 + j\omega \frac{R_1 R_2}{R_1+R_2} C_1} \quad (1)$$

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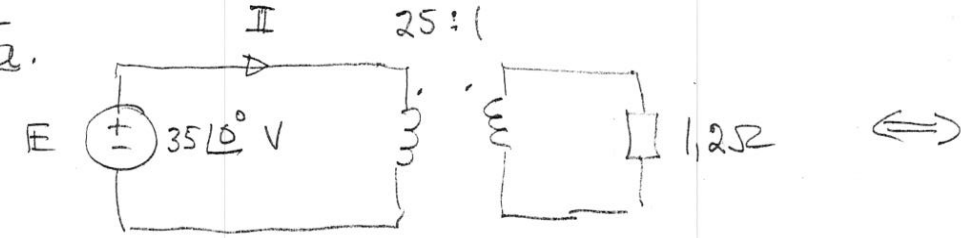
$$= \frac{R_3}{1 + j\omega R_3 C_2} \cdot (1) \Rightarrow$$

$$V_{ut} = -\frac{R_3}{R_2} \cdot \frac{1}{1 + j\omega R_3 C_2} \cdot V_a = -\frac{R_3}{R_2} \cdot \frac{j\omega \frac{R_1 R_2}{R_1+R_2} C_1}{(1 + j\omega R_3 C_2)\left(1 + j\omega \frac{R_1 R_2}{R_1+R_2} C_1\right)} =$$

$$= -10 \cdot \frac{j\frac{\omega}{500}}{\left(1 + j\frac{\omega}{2\cdot 10^5}\right)\left(1 + j\frac{\omega}{500}\right)} \Rightarrow$$

$K = -10$ $\omega_1 = 500$ rad/s $\omega_2 = 200\,000$ rad/s

**5a.**

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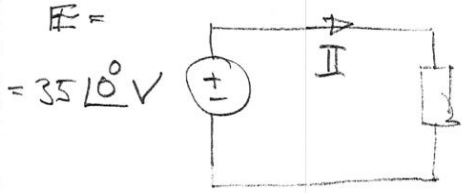
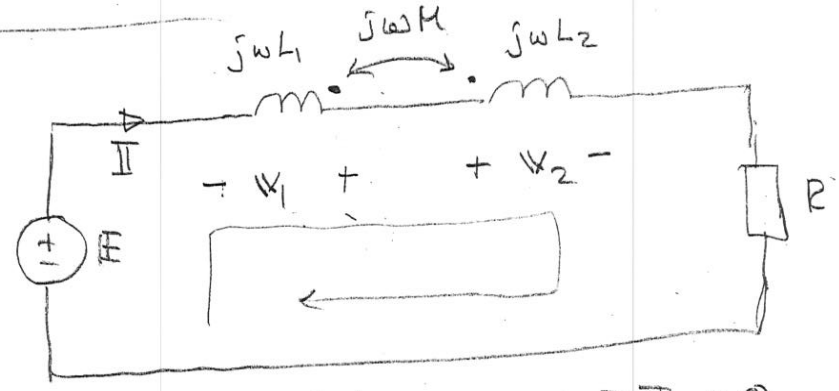



$25^2 \cdot 1{,}2\,\Omega = 750\,\Omega \Rightarrow$

$$\text{II} = \frac{35\angle 0^\circ}{750\,\Omega} = 46{,}7\angle 0^\circ \text{ mA} \Rightarrow \underline{i(t) = 46{,}7 \cdot \cos 1000t \text{ mA}}$$

**5b.**




Kirchhoff: $+E + V_1 - V_2 - R\,\text{II} = 0$

$\left.\begin{aligned} \cdot\ V_1 &= -j\omega L_1\,\text{II} + j\omega M\,\text{II} \\ \cdot\ V_2 &= j\omega L_2\,\text{II} - j\omega M\,\text{II} \end{aligned}\right\} \Rightarrow$

$$E - j\omega L_1\,\text{II} + j\omega M\,\text{II} - j\omega L_2\,\text{II} + j\omega M\,\text{II} - R\,\text{II} = 0 \Rightarrow$$

$$E = (j\omega L_1 + j\omega L_2 - 2j\omega M + R)\cdot \text{II} =$$

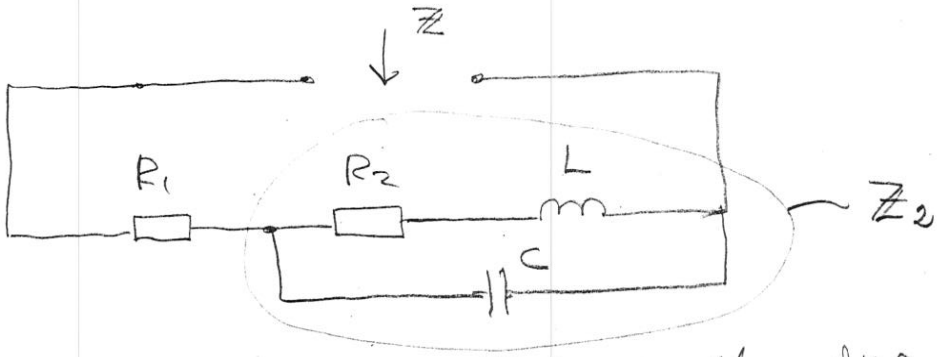
$$= (j10 + j5 - j8 + 10)\,\text{II} = (10 + j7)\,\Omega \cdot \text{II} =$$

$$= 7{,}68\angle 35{,}0^\circ\,\Omega \cdot \text{II} \Rightarrow$$

$$\text{II} = \frac{35\angle 0^\circ\,\text{V}}{7{,}68\angle 35{,}0^\circ\,\Omega} = 4{,}56\angle -35{,}0^\circ\ \text{A} \Rightarrow$$

$$\underline{i(t) = 4{,}56 \cdot \cos(1000t - 35{,}0^\circ)\ \text{A}}$$

6.




Resonans: Resonans om $\underline{Z}$ är reell, dvs om $\underline{Z}_2$ är reell.

$$\underline{Z}_2 = \frac{1}{j\omega C} /\!/ (R_2 + j\omega L) = \frac{\frac{1}{j\omega C}(R_2 + j\omega L)}{\frac{1}{j\omega C} + R_2 + j\omega L} =$$

$$= \frac{R_2 + j\omega L}{1 + j\omega R_2 C - \omega^2 LC} = \frac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C} \Rightarrow$$

$\underline{Z}_2$ reell om

$$\frac{R_2}{1-\omega^2 LC} = \frac{\omega L}{\omega R_2 C} \Rightarrow 1 - \omega^2 LC = \frac{R_2^2 C}{L} \Rightarrow$$

$$\omega = \frac{1}{\sqrt{LC}} \cdot \sqrt{1 - \frac{R_2^2 C}{L}} = \underline{122{,}5 \text{ krad/s}}$$

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Resonans saknas om

$$1 - \frac{R_2^2 C}{L} < 0 \iff C > \frac{L}{R_2^2} = \underline{25 \text{ nF}}$$

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$\underline{Z}$ vid resonans

$$\underline{Z}_{res} = R_1 + \underline{Z}_{2res} = R_1 + \frac{L}{R_2 C} = \underline{1800\,\Omega}$$

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