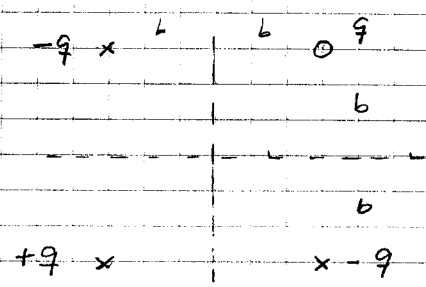
Lösningar till Elektromagnetiska fält för E3

den 17/4 2004 EP

1) Se Cheng eller Elfält i sammandrag

2) q på ledaren ⇒ −q på metallplanen. Spegla!

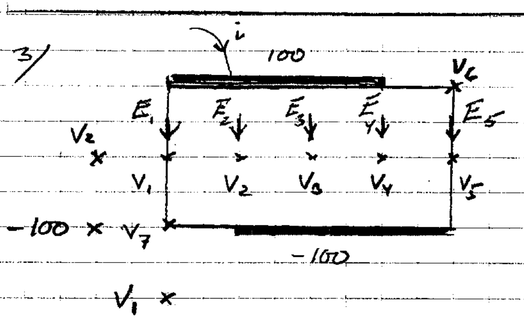


Potential på ledaren

$$V_q = \frac{q}{2\pi\varepsilon_0 l}\left(\ln\frac{2b}{a} + \ln\frac{2b}{2\sqrt{2}b}\right) = \frac{q}{2\pi\varepsilon_0 l}\ln\frac{2b}{\sqrt{2}a}\ ; \quad V_{plan}=0$$

$$C = \frac{q}{V_q - V_{plan}} = \frac{2\pi\varepsilon_0 l}{\ln\frac{\sqrt{2}b}{a}}$$

3)



Ansätt potentialer utanför skivan, så att E blir tangentiell vid gränsplan. Pga symmetri $V_3=0$, $V_4=-V_2$, $V_5=-V_1$; $V_7=-V_6$

a) $V_1 = \frac{1}{4}(100+V_2+V_7+V_2)$; $V_2 = \frac{1}{4}(100+V_1-100+V_3)$; $V_7 = \frac{1}{4}(V_1-100+V_1-100)$

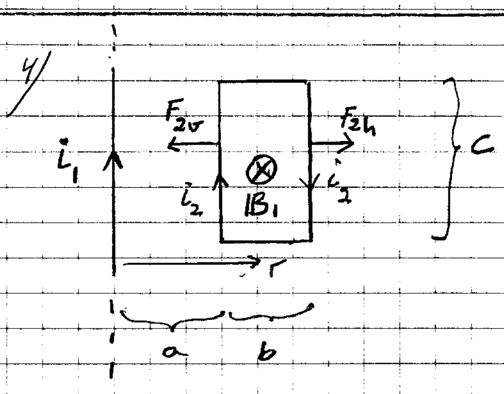
⇒ $V_1 = \frac{50}{3} = -V_5$; $V_2 = \frac{25}{6} = -V_4$; $V_3 = 0$; $V_6 = \frac{125}{3} = -V_7$

b) E-fält $E \approx \frac{\Delta V}{h}$; $E_1 = \frac{100-V_1}{h}$; $E_2 = \frac{100-V_2}{h}$; $E_3 = \frac{100-V_3}{h}$; $E_4 = \frac{100-V_4}{h}$

$E_5 = \frac{V_6-V_5}{h}$; ström $i = \sigma\left[E_1\frac{h}{2}d + E_2hd + E_3hd + E_4hd + E_5\frac{h}{2}d\right]$

⇒ $i = \sigma d\cdot 370{,}8$ ⇒ $R = \frac{100-(-100)}{i} = 1348\ \Omega$ (datorberäkn. $1520\ \Omega$)

4)



Kraft på de båda c-ledarna

$$\mathbf{F}_2 = \int i_2\, d\mathbf{l}_2 \times \mathbf{B}_1 \ ; \quad B_1 = \frac{\mu_0 i_1}{2\pi r}$$

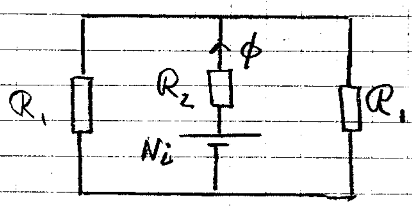
Vänstra $F_{2v} = \frac{\mu_0 i_1 i_2 c}{2\pi a}$

högra $F_{2h} = \frac{\mu_0 i_1 i_2 c}{2\pi(a+b)}$

Kraften på b-ledarna: lika stora men motriktade ⇒ $\Sigma F = 0$ där

$$F_{2tot} = F_{2v} - F_{2h} = \frac{\mu_0 i_1 i_2 c}{2\pi}\left(\frac{1}{a} - \frac{1}{a+b}\right)$$ attraktionskraft

5/ Induktans $L = \frac{N\phi}{i}$; Ekvivalent el. krets:



$$\phi = \frac{Ni}{\mathcal{R}_2 + \mathcal{R}_1 /\!/ \mathcal{R}_1} = \frac{Ni}{\mathcal{R}_2 + \mathcal{R}_1/2}$$

reluktanser

$$\mathcal{R} = \frac{\ell}{\mu S} \qquad \mathcal{R}_1 = \frac{0{,}25}{250 \cdot 4\pi \cdot 10^{-7} \cdot 4 \cdot 10^{-4}}$$

$$\mathcal{R}_2 = \frac{0{,}10}{250 \cdot 4\pi \cdot 10^{-7} \cdot 4 \cdot 10^{-4}} \Rightarrow \mathcal{R}_2 + \frac{\mathcal{R}_1}{2} = 1{,}79 \cdot 10^6$$

$$\Rightarrow L = \underline{\underline{0{,}140 \text{ H}}}$$

6/ $\frac{\sigma}{\omega\varepsilon} = \frac{10^7 \cdot 36\pi}{2\pi \cdot 10^4 \cdot 10^{-9}} \gg 1$ approximera i γ och Z!

$$\Rightarrow \gamma = \sqrt{j\omega\mu_0\sigma} = (1+j)\sqrt{\frac{\omega\sigma\mu_0}{2}} = 200\pi(1+j) = 628{,}3(1+j) = \alpha + j\beta$$

$$Z = \frac{j\omega\mu_0}{\gamma} = \frac{4\pi \cdot 10^{-5}}{1-j} = \frac{4\pi}{\sqrt{2}} \cdot 10^{-5} e^{j\pi/4}$$

$\Rightarrow$ komplexa fälten

$$\begin{cases} \mathbf{E}(z) = \hat{x} E_0 e^{j\pi/6} e^{-\gamma z} \\ \mathbf{H} = \frac{1}{Z} \underbrace{\hat{k}}_{\hat{z}} \times \mathbf{E} = \hat{y} \frac{E_0}{Z} e^{j\pi/6} e^{-\gamma z} \end{cases}$$

tidsuttrycken

$$\begin{cases} \mathbf{E}(z,t) = \hat{x} E_0 e^{-628{,}3 z} \cos\left(2 \cdot 10^4 \pi t - 628{,}3 z + \underbrace{\frac{\pi}{6}}_{30°}\right) \text{ V/m} \\ \mathbf{H}(z,t) = \hat{y}\, 11{,}2 \cdot 10^3 E_0 e^{-628{,}3 z} \cos\left(2 \cdot 10^4 \pi t - 628{,}3 z - \underbrace{\frac{\pi}{12}}_{15°}\right) \text{ A/m} \end{cases}$$